6

SPUR GEARS

Gears are toothed wheels used to transmit mechanical energy accurately, in the form of rotational movement from one member to another.

Classification of gears : The gears may be classified as follows :

1. *According to the relative position of the axes of the shaft on which the gears are mounted :*

Parallel axes	Intersecting axes	Non-intersecting and non-parallel axes
1. Spur gear 2. Helical gear 3. Double helical gear	1. Bevel gears a) Straight bevel b) Spiral bevel c) Zerol bevel	1. Crossed helical gear 2. Worm gear 3. Hypoid gear

2. *According to the type of gearing :*
   a) External
   b) Internal

In external gearing, the gears mesh externally with each other and they rotate in opposite direction. In internal gearing, the gears mesh internally with each other and they rotate in the same direction.

3. *According to the peripheral velocity of the gears :*
   a) Low velocity gears ($V < 3$ m/s)
   b) Medium velocity gears ($3 \leq V \leq 15$ m/s)
   c) High velocity gears ($V > 15$ m/s)
4. *According to the position of teeth on the gear surface :*
   a) Straight
   b) Inclined
   c) Curved

Advantages and disadvantages of gear drives

Advantages :

1. It is a positive drive and is used to connect closely spaced shafts.
2. High efficiency, compactness, reliable service, more life, simple operation and low maintenance.
3. It can transmit heavier loads than other drives and can be used where precise timing is desired.

Disadvantages :

1. Not suitable for large center distances because the drive becomes bulky.
2. High production cost.
3. Due to errors and inaccuracies in their manufacture, the drive become noisy.

Law of gearing

When the tooth profiles are designed so as to produce a constant angular velocity ratio during meshing they are said to have *conjugate action.* One of the tooth profile which gives the conjugate action is the involute profile.

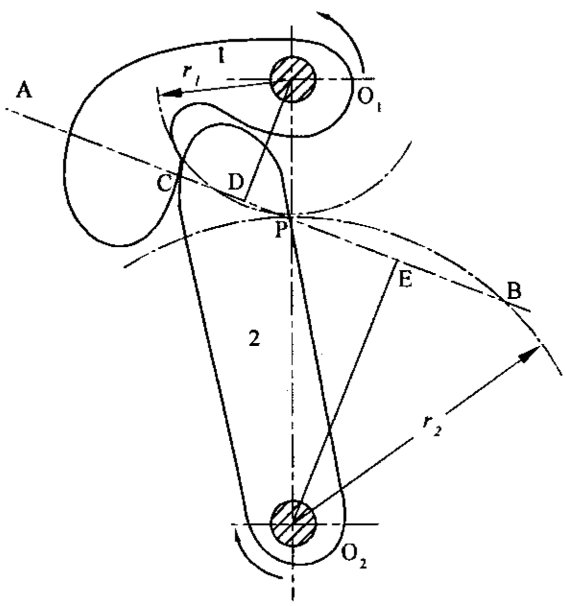

Fig. 6.1

When one curve surface pushes against another, the point of contact occurs where the two surfaces are tangent to each other at C as in fig. 6.1 and the forces at any instant are directed along the common normal AB to the two curves. The line AB, representing the direction of action of the forces. is called *line of action.* The line of action (pressure line) will intersect the line of centers O_1 O_2 at point P known as *pitch point.* Pure rolling exist at pitch point. Circles drawn through point P from each center are called pitch circles.

Let O_1 D and O_2 E be perpendiculars to the common normal. Triangles O_1PD and O_2PE are similar.

$$\therefore \qquad \frac{O_1P}{O_1D} = \frac{O_2P}{O_2E}$$

or

$$\frac{O_2E}{O_1D} = \frac{O_2P}{O_1P} = \frac{r_2}{r_1} = \frac{\omega_1}{\omega_2}$$

The velocity ratio can only be constant if the common normal AB passes through P. *The law of gearing states that in order to transmit motion at constant angular velocity ratio, the pitch point must remain fixed, that is, all the lines of action for every instantaneous point of contact must pass through the same point P.*

Velocity of sliding : The velocity of sliding is equal to the product of the sum of angular velocities and the distance from the point of contact to the point of intersection of the common normal and the line joining the centers of rotation.

Thus velocity of sliding $= (\omega_1 + \omega_2) \times CP$

Gear tooth profiles

The commonly used forms of teeth which satisfy the law of gearing are the following :

(1) Involute and (2) Cycloid.

Involute : An involute may be defined as the locus of a point on a straight line which rolls without slipping, on the circumference of a circle. An involute is generated by the end of a string being unwound from a cylinder or by a point on a line as the line rolls on the circumference of a circle without slipping. The characteristic of an involute is that any normal to an involute is tangent to the base circle from which the involute is generated.

Properties of involute tooth profile :

1. A normal to an involute is a tangent to the base circle.
2. When involutes are in mesh, then the pressure angle remain constant.
3. The involute is the only tooth form which is insensitive to the center distance of its base circle.
4. The shape of the involute profile depends only on the dimension of the base circle.
5. The radius of curvature of an involute is equal to the length of tangent to the base circle.
6. Basic rack for involute tooth profile has straight line form.
7. The common tangent to the base circles of the two involutes is the line of action and also the path of contact between the involutes.
8. When two involutes are in mesh, then they transmit constant angular velocity ratio and it is inversely proportional to the radius of base circles.
9. Manufacturing is easy due to single curvature.

10. Involute gears in mesh give conjugate action.
11. Suitable for motion and power transmission.

Cycloid

A cycloid is the locus of a point on the circumference of a circle which rolls, without slipping, along a straight line. The faces of the teeth are epicycloid generated on the pitch circles and the flanks are hypocycloids generated inside the pitch circles.

Comparison between involute and cycloidal tooth forms

Feature	Involute	Cycloidal
Basis form	Constant for all gears	Variable depending upon the gear ratio
Strength	Less	Good strength
Angular velocity at varying center distance	Constant	Variable
Tooth contact	Line bedding on a concave face	Generally area with a convex face
Pressure angle at a given center distance	Constant	Variable
Path of contact	Straight line	Curve
Standardization of tooth form	Easy	Difficult
Economy of tooling equipment	Good	Poor
Interference	May exist accordingly	No interference
Wear	More	Less
Manufacturing	Easy	Difficult

Spur gear

This the simplest form of gears for transmitting motion between two parallel shafts. The teeth are straight and parallel with the direction of rotation.

Spur gear terminology *(Refer fig. 6.2)*

Pitch circle : The pitch circle is a theoretical circle upon which all calculations are usually based. The pitch circles of a pair of mating gears are tangent to each other.

Pitch diameter : The pitch diameter d is the diameter of the pitch circle.

Circular pitch : Circular pitch p is the distance, measured on the pitch circle from a point on one tooth to a corresponding point on an adjacent tooth. Thus the circular pitch is equal to the sum of the tooth thickness and the width of space.

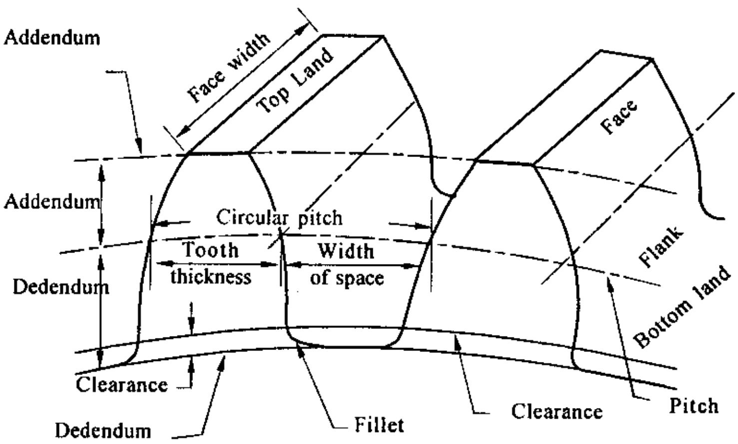

Fig. 6.2

i.e,
$$p = \frac{\pi d}{z}$$

where d = Pitch circle diameter

z = Number of teeth.

Tooth thickness : It is the thickness of tooth measured along the pitch circle. For standard gear, without back lash,

Tooth thickness $t = \frac{p}{2}$

Width of tooth space : It is the width of the space between tooth measured along the pitch circle.

Gear and pinion : A gear is the larger of the two mating toothed wheels and the smaller of the two is the pinion.

Rack : A rack is a portion of a gear of infinite radius.

Angular velocity ratio : The angular velocity ratio for a pair of spur gear is inversely proportional to their pitch circle radii or their number of teeth.

i.e.,
$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{z_2}{z_1}$$

where ω_1 = Angular velocity of the driver (usually pinion)

ω_2 = Angular velocity of the follower (gear)

r_1 = Pitch circle radius of the driver

r_2 = Pitch circle radius of the follower

z_1 = Number of teeth on the driver

z_2 = Number of teeth on the follower

Center distance : The center distance c is the distance from center to center for two mating gears.

i.e., $$c = r_1 + r_2$$

where r_1 and r_2 are the pitch circle radii of the mating gears.

Module : The module m expressed in millimeters is the ratio of the pitch diameter d to the number of teeth z.

i.e., $$\text{Module } m = \frac{d}{z} = \frac{p}{\pi}$$

Diametrical pitch : The diametral pitch P is the ratio of the number of teeth on the gear to the pitch diameter. It is the reciprocal of the module.

The relationship between the circular pitch and the diametrical pitch is $P \times p = \pi$

Top land : It is the surface bounded by the sides of the gear and active profiles.

Bottom land (Root land) : It is the surface bounded by fillets of the adjacent teeth and sides of the gear blank.

Face of the tooth : It is the part of the tooth surface lying above the pitch surface.

Flank of the tooth : It is the part of the tooth surface lying below the pitch surface.

Addendum : The addendum a is the radial distance between the top land and pitch surface.

Dedendum : The dedendum b is the radial distance from the bottom land to the pitch circle.

The Whole depth : It is the sum of the addendum and dedendum.

The Working depth : It is the depth of engagement of a pair of gears, that is, the sum of the addendums.

Addendum circle : It is a circle bounding the top of the teeth.

Dedendum circle : It is a circle passing through the roots of all the teeth.

Clearance circle : It is a circle that is tangent to the addendum circle of mating gears.

Base circle : It is an imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.

Base circle radius $r_b = r \cos \phi$

where r = pitch circle radius

ϕ = pressure angle

Base pitch p_b : It is the distance measured along the base circle from a point on one tooth to the corresponding point on an adjacent tooth.

$$p_b = p \cos \phi$$

Clearance : The clearance is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

Backlash : It is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circle.

Pressure angle ϕ: It is the angle which the common normal to two teeth at the point of contact makes with the common tangent to the two pitch circles at the pitch point.

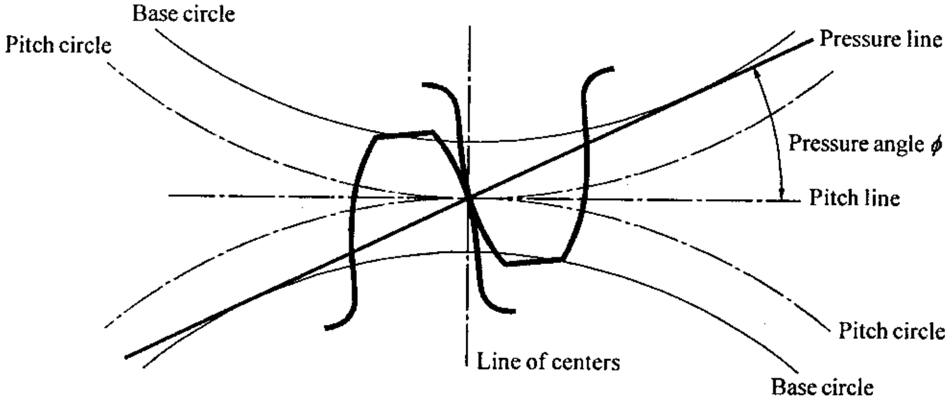

Fig. 6.3

The line normal to the surface of the teeth at the contact point is termed the pressure line or line of contact. The line tangent to the pitch circles is termed the pitch line. Therefore, the pressure angle is measured between the pitch line and the pressure line as illustrated in fig. 6.3.

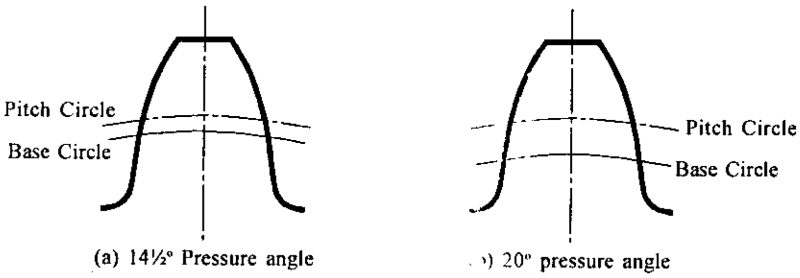

Fig. 6.4

Gears are manufactured in a wide range of press re angle. Most gears are standardized at 14½° and 20° pressure angle. The pressure angle aff cts the relative shape of a gear tooth as shown in fig. 6.4.

Initial point of contact : Fig. 6.5 shows two matting t th at three intervals of the engagement process. The point of initial contact is at the intersection f the driven gear's addendum circle with the pressure line. i.e., point A of fig. 6.5.

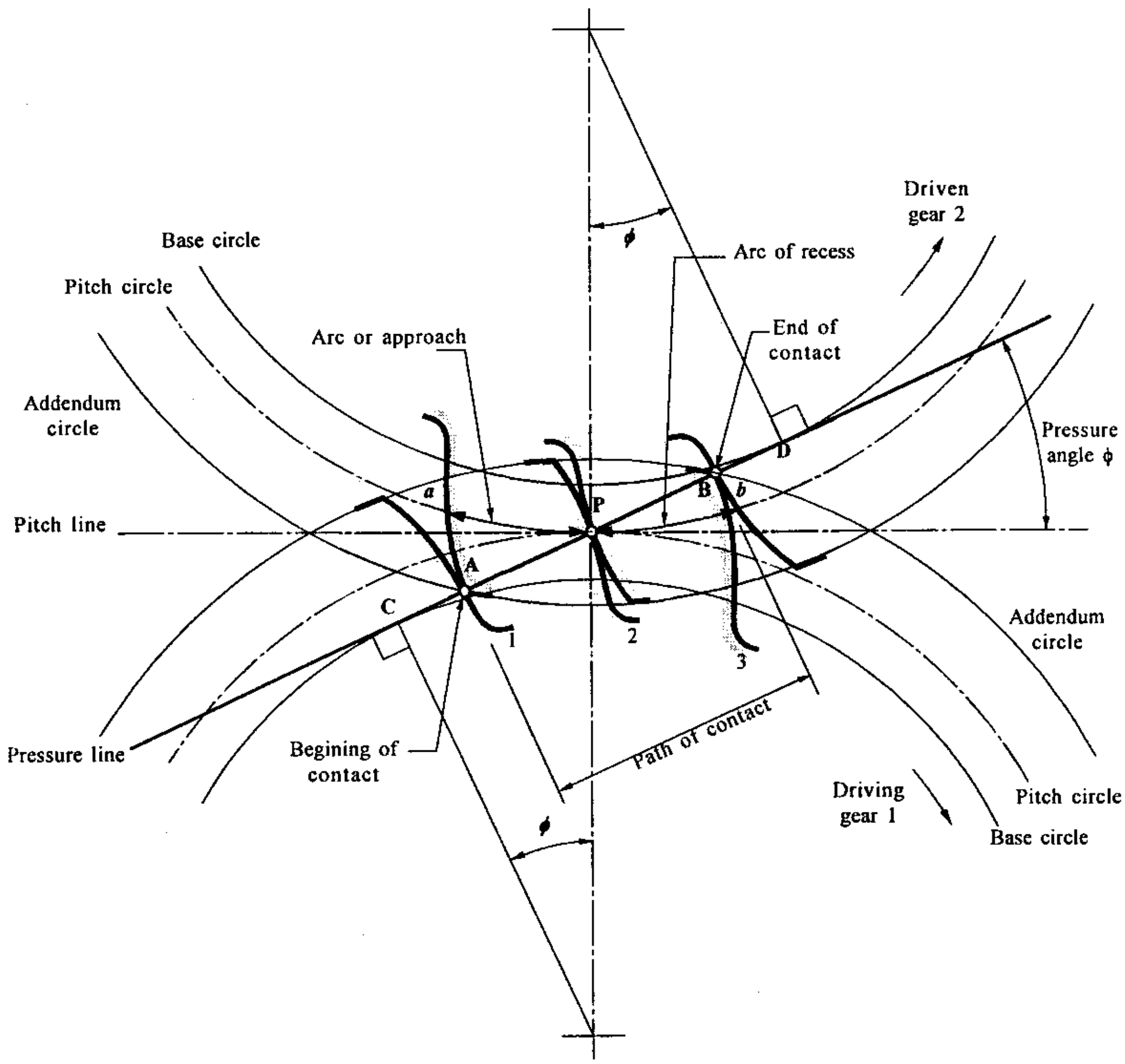

Fig. 6.5

Final point of contact: The point of final contact occurs at the intersection of the driver gear's addendum circle with the pressure line. i.e., point B of fig. 6.5.

Path of contact: It is a curve traced out by the point of contact of two teeth from the beginning to the end of engagement. i.e., line AB of fig. 6.5.

Arc of contact: Arc of contact is the arc on the pitch circle through which a tooth profile moves from the begining to the end of contact with a mating profile. The arc of the pitch circle through which a tooth profile moves from its begining of contact until the point of contact arrives at the pitch point is called the *arc of approach* (arc *ap*). The arc of the pitch circle through which a tooth profile moves from contact at pitch point until the contact ends is called the *arc of recess* (arc *pb*). The sum of these two arc length is equal to the arc of contact i.e., arc *ab*.

Arc of contact ab = arc of approach ap + arc of recess pb

$$= \frac{\text{Path of contact AB}}{\cos\phi}$$

Contact ratio : It is a number which indicates the average number of pairs of teeth in contact. It is equal to the length of the path of contact divided by the base pitch or the ratio of arc of contact to circular pitch.

$$\text{Contact ratio} = \frac{\text{Path of contact}}{\text{Base pitch}} = \frac{\text{Path of contact}}{p\cos\phi} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

Length of arc of contact

Referring to fig. 6.6 the addendum circles cut the common tangent CD at the points A and B. In other words, contact of the tooth begins at A (where the addendum of driven gear 2 intersects pressure line) and ends at points B (where the addendum of driver gear 1 intersects the pressure line).

Let
O_1P = Pitch circle radius of pinion (dirver) = r_1
O_2P = Pitch circle radius of gear (follower) = r_2
O_1C = Base circle radius of pinion = r_{b_1}
O_2D = Base circle radius of gear = r_{b_2}
O_1B = Addendum circle radius of pinion = r_{a_1}
O_2A = Addendum circle radius of gear = r_{a_2}
AP = Path of approach = A
PB = Path of recess = R

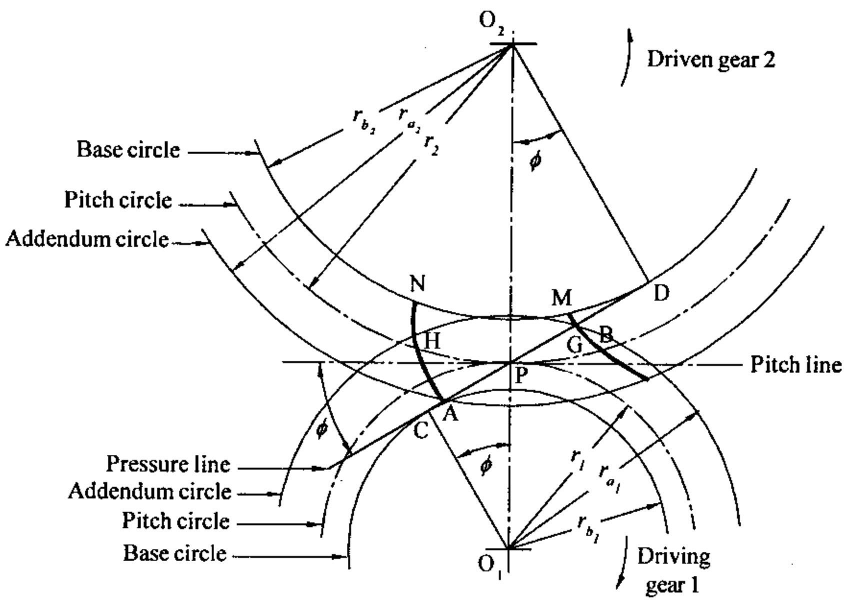

Fig. 6.6

Path of contact $AB = AP + PB = A + R$ ......(1)

From the fig. 6.6.

Path of approach $A = AP = AD - PD$ ......(2)

From triangle O_2PD,

$$PD = O_2P \sin\phi = r_2 \sin\phi \quad ......(3)$$

and $O_2D = r_{b2} = O_2P \cos\phi = r_2 \cos\phi$

From triangle O_2AD,

$$AD^2 = O_2A^2 - O_2D^2 = r_{a2}^2 - r_2^2 \cos^2\phi$$

$$\therefore AD = \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} \quad ......(4)$$

Substituting the value of AD and PD in equation (2), we get

$$\text{Path of approach } A = \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} - r_2 \sin\phi \quad ......(5)$$

Similar, path of recess $R = PB = BC - PC$ ......(6)

From triangle O_1PC

$$PC = O_1P \sin\phi = r_1 \sin\phi \quad ......(7)$$

and $O_1C = r_{b1} = O_1P \cos\phi = r_1 \cos\phi$

From triangle O_1BC,

$$BC^2 = O_1B^2 - O_1C^2 = r_{a_1}^2 - r_1^2 \cos^2\phi$$

$$\therefore BC = \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} \quad ......(8)$$

Substituting the values of BC and PC in equation (6), we get

$$\text{Path of recess } R = \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - r_1 \sin\phi \quad ......(9)$$

Substituting the values of A and R in equation (1), we get

$$\text{Path of contact} = \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} + \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - (r_1 + r_2)\sin\phi \quad ......(10)$$

If the diameters of the driver and driven gears are same, then equation (10) becomes

$$\text{Path of contact} = 2\sqrt{r_a^2 - r^2 \cos^2\phi} - 2r \sin\phi \quad ......(11)$$

where $r_{a_1} = r_{a_2} = r_a$, and $r_1 = r_2 = r$

The values of path of approach A and the path of recess R, are valid only for the conditions.

$$A \le r_1 \sin\phi \text{ and}$$

$$R \le r_2 \sin\phi$$

because the contact cannot begin before the point C or end after the point D.

i.e., the maximum path of approach $A = r_1 \sin\phi$

The maximum path of recess $R = r_2 \sin\phi$

$$\text{Arc of contact} = \text{arc GH} = \frac{\text{arc MN}}{\cos\phi} \qquad \text{......(12)}$$

Since the involutes GM and HN may be generated by rolling the tangent CD on the base circle,

arc MN = path of contact AB

∴ Equation (12) becomes

$$\text{Arc of contact} = \frac{\text{path of contact}}{\cos\phi} \qquad \text{......(13)}$$

In this chapter, suffix 1 represents the pinion and suffix 2 represents the gear, irrespective of whether it is a driver or a follower. In most of the applications, the pinion will be the driver. In case the gear wheel is the driver, the formula for the path of approach and path of recess have to be interchanged.

Interference

The contact of portions of tooth profiles which are not conjugate is called *interference*. Consider the driver gear 1 turns clockwise as shown in fig. 6.7. The initial and final points of contact are designated A and B respectively, and are located on the pressure line. The points of tangency of the pressure line with the base circle C and D are located inside of points A and B. Interference is present. The interference is explained as follows. Contact begins where the tip of the driven tooth contacts the flank of the driving tooth. In this case the flank of the driving tooth

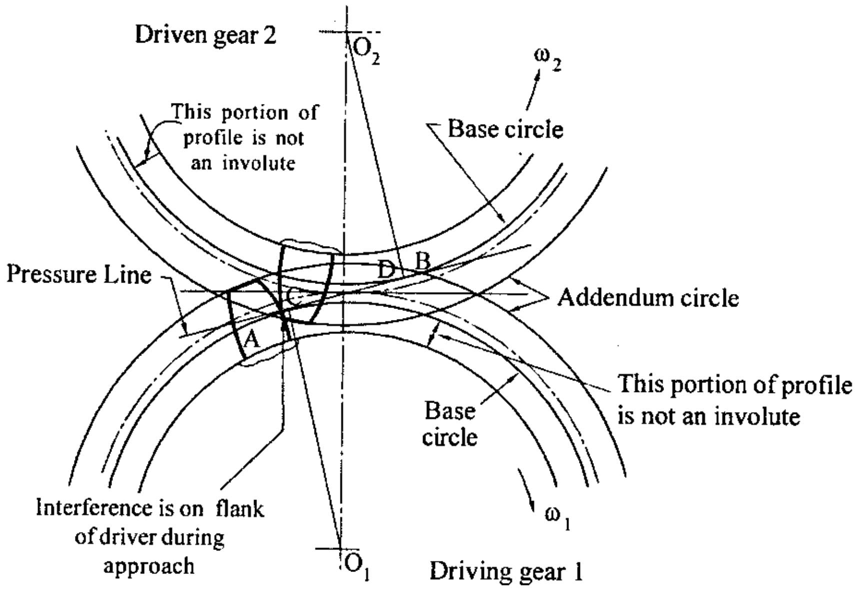

Fig. 6.7

first makes contact with the driven tooth at point A, and this occurs before the involute portion of the driving tooth comes within range. The actual effect is that the involute tip or face of the driven gear tends to dig out the non-involute flank of the driver. Same effect occurs again as the teeth leave contact. Contact should end at point D or before. Since it does not end until point B, the effect is for the tip of the driving tooth to dig out or interfere with the flank of the driven tooth.

Methods of avoiding interference and its effect

Interference between the tips of the gear teeth and the flanks of the pinion teeth may be avoided by one of the following methods.

1. *Under cutting* : Under cutting the flanks of the pinion teeth leads to a weakening of the tooth and complication of manufacture.
2. *Increasing the center distance* : The correct tooth action is maintained but the pressure angle is increased, leading to higher tooth pressures and increased backlash.
3. *By tooth correction (modifying the gear and pinion addenda)* : The pressure angle,center distance and base circles remain unaltered but thickness of gear tooth at the pitch circle becomes greater than $p/2$ and that of the pinion becomes less than $p/2$.

Minimum number of teeth required to avoid interference

A pinion 1 turns counter-clockwise and drives a gear as shown in fig. 6.8. The common tangent to the base circle is CD. The points C and D are called interference points, and if the path of contact does not extend beyond either of these points, interference is avoided. The limiting value of the radius of the addendum circle of the pinion is O_1 C and the gear is O_2 D. To determine whether there will be interference when two gears are in mesh, each addendum circle radius is compared with its limiting value. The interference is more likely to occur on the pinion than on the gear, and in this case, the critical radius is O_2 D which limits the number of teeth on pinion.

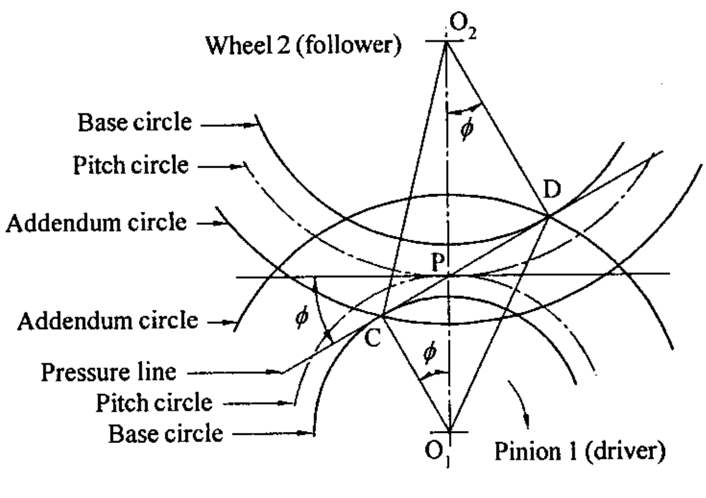

Fig. 6.8

Let $O_1 P$ = Pitch circle radius of pinion = r_1

$O_2 P$ = Pitch circle radius of gear = r_2

$O_2 D$ = Base circle radius of gear = r_{b_2}

$O_2 C$ = Addendum circle radius of gear = r_{a_2}

From the right angle-triangle $O_2 C D$,

$$O_2 C^2 = O_2 D^2 + CD^2 = O_2 D^2 + (C P + P D)^2$$
$$= O_2 D^2 + (O_1 P \sin\phi + O_2 P \sin\phi)^2$$

i.e.,
$$r_{a_2}^2 = r_{b2}^2 + (r_1 \sin\phi + r_2 \sin\phi)^2$$
$$= (r_2 \cos\phi)^2 + (r_1 + r_2)^2 \sin^2\phi \qquad (\because r_{b2} = r_2 \cos\phi)$$
$$= r_2^2 \cos^2\phi + r_1^2 \sin^2\phi + r_2^2 \sin^2\phi + 2 r_1 r_2 \sin^2\phi$$
$$= r_2^2 (\cos^2\phi + \sin^2\phi) + r_1^2 \sin^2\phi + 2 r_1 r_2 \sin^2\phi$$
$$= r_2^2 + r_1^2 \sin^2\phi + 2 r_1 r_2 \sin^2\phi$$

$\therefore$ Maximum addendum circle radius of the gear to avoid interference is

$$r_{a_2} = \sqrt{r_2^2 + r_1^2 \sin^2\phi + 2 r_1 r_2 \sin^2\phi} \qquad \text{......(1)}$$

Minimum number of teeth :

We know that
$$r_{a_2}^2 = r_2^2 + r_1^2 \sin^2\phi + 2 r_1 r_2 \sin^2\phi$$
$$= r_2^2 \left[1 + \frac{r_1^2}{r_2^2} \sin^2\phi + \frac{2r_1}{r_2} \sin^2\phi\right]$$
$$= r_2^2 \left[1 + \frac{r_1}{r_2} \sin^2\phi \left(\frac{r_1}{r_2} + 2\right)\right] \qquad \text{......(2)}$$

$$\text{Gear ratio } i = \frac{n_1}{n_2} = \frac{r_2}{r_1} = \frac{z_2}{z_1}$$

$\therefore$ Equation (2) becomes

$$r_{a_2}^2 = r_2^2 \left[1 + \frac{\sin^2\phi}{i}\left(\frac{1}{i} + 2\right)\right]$$

or
$$r_{a_2} = r_2 \qquad \text{......(3)}$$

Addendum of gear $a_2 = a_g\, m = r_{a_2} - r_2$

where a_g is called addendum coefficient for the gear.

$$\therefore a_2 = a_g\, m = r_2 \left[1 + \frac{\sin^2\phi}{i}\left(\frac{1}{i} + 2\right)\right]^{1/2} - r_2$$

$$a_g m = r_2 \left[\left\{ 1 + \frac{\sin^2 \phi}{i} \left(\frac{1}{i} + 2 \right) \right\}^{\frac{1}{2}} - 1 \right]$$

$$= \frac{m z_2}{2} \left[\left\{ 1 + \frac{\sin^2 \phi}{i} \left(\frac{1}{i} + 2 \right) \right\}^{\frac{1}{2}} - 1 \right] \qquad \left(\because r_2 = \frac{m z_2}{2} \right)$$

or

$$a_g = \frac{z_2}{2} \left[\left\{ 1 + \frac{\sin^2 \phi}{i} \left(\frac{1}{i} + 2 \right) \right\}^{\frac{1}{2}} - 1 \right]$$

∴ Number of teeth on gear to avoid interference is

$$z_2 = \frac{2 a_g}{\sqrt{1 + \frac{\sin^2 \phi}{i} \left(\frac{1}{i} + 2 \right)} - 1} \qquad \text{......(4)}$$

Number of teeth on pinion $z_1 = \frac{z_2}{i}$ ......(5)

If the number of teeth on pinion z_1 is the same as the number of teeth on gear z_2, then the gear ratio i is equal to one.

∴ Equation (4) becomes

$$z_1 = z_2 = \frac{2 a_g}{\sqrt{1 + 3 \sin^2 \phi} - 1} \qquad \text{......(6)}$$

Involute pinion and rack

Fig.6.9 shows a pinion driving a rack. Rack is merely a spur gear, the radius of whose pitch circle has become infinite. The sides of involute teeth on a rack are straight lines making an angle to the line of centers equal to the pressure angle.

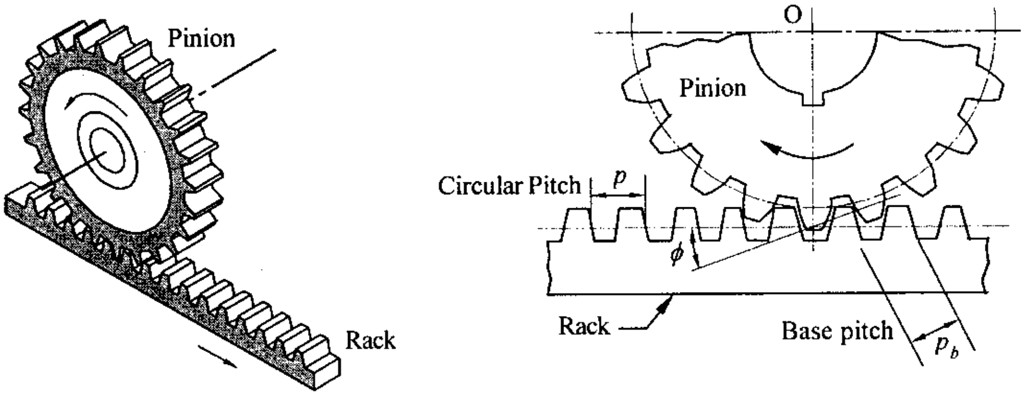

Fig. 6.9

Length of path of contact for rack on pinion

Referring to fig. 6.10, and assuming the pinion to be the driver, the addendum of the rack cuts the pressure line at point A and the addendum circle of pinion cuts the pressure line at point B. i.e., contact of tooth begins at A and ends at B. The pitch circle of the pinion and the pitch line of the rack are tangent to each other at P.

$\therefore$ Path of contact AB = Path of approach AP + Path of recess PB ......(1)

Path of approach $A = AP = \dfrac{a}{\sin\phi}$ ......(2)

where a is the addeudum of the rack.

Path of recess $R = PB = BC - PC$ ......(3)

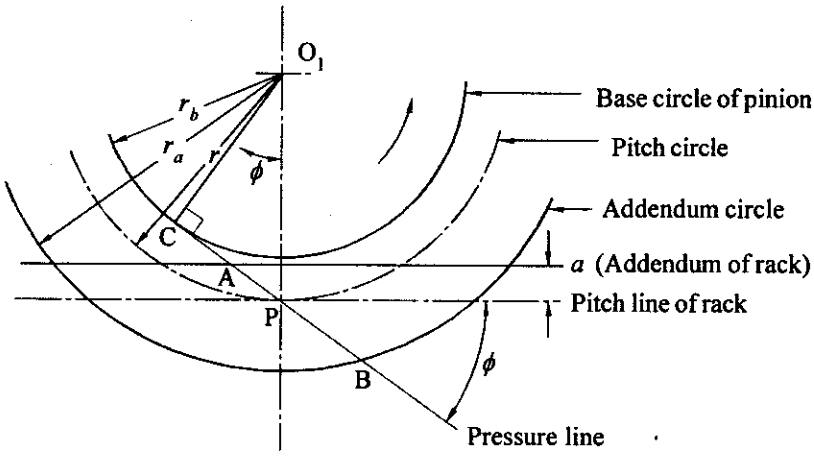

Fig. 6.10

From triangle O_1 PC,

$$PC = O_1 P \sin\phi = r \sin\phi \quad ......(4)$$

and $\;{}_{\text{ʢ}}C = r_b = O_1 P \cos\phi = r\cos\phi$

From triangle O_1 BC,

$$BC^2 = O_1B^2 - O_1C^2$$

$$\text{i.e., } BC^2 = r_a^{\,2} - r^2\cos^2\phi \quad ......(5)$$

Substitute the values of PC and BC in equation (3), we get

Path of recess $R = \sqrt{r_a^2 - r^2\cos\phi} - r\sin\phi$ ......(6)

Substituting the values of A and R in equation (1), we get

Path of contact $= \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi + \dfrac{a}{\sin\phi}$ ......(7)

Minimum number of teeth for pinion on rack

Referring fig. 6.11

Let $a = a_r m$ = Addendum of rack = CN

where a_r is the addendum coefficient of the rack.

From triangles, PCN and O_1 PC

$$CN = PC \sin\phi \text{ and } PC = O_1P \sin\phi$$

$$\therefore CN = O_1P \sin^2\phi = r\sin^2\phi = a \qquad \text{......(1)}$$

but O_1P = pitch circle radius of pinion $r = \frac{mz}{2}$

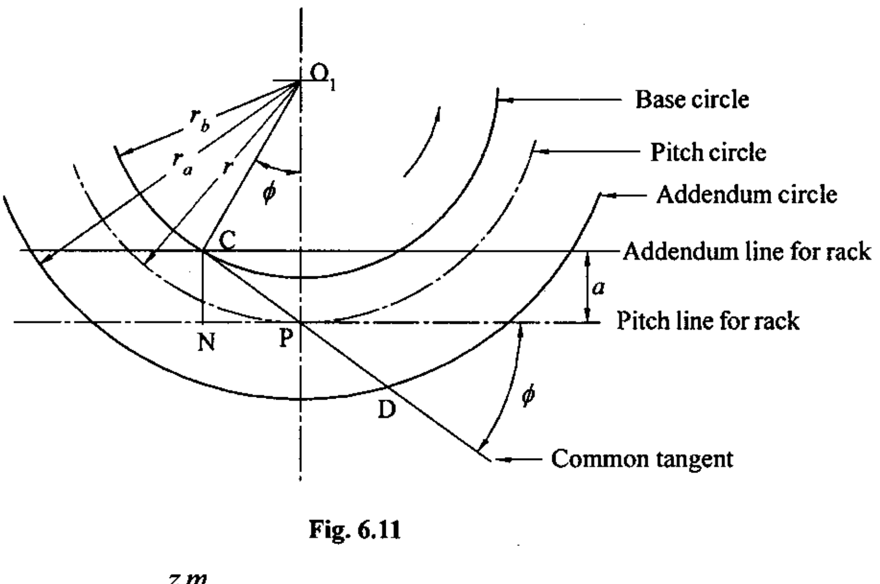

Fig. 6.11

$$\therefore CN = \frac{zm}{2}\sin^2\phi = a_r m$$

$$\text{Number of teeth on pinion } z = \frac{2a_r}{\sin^2\phi} \qquad \text{......(2)}$$

Internal gear

Fig. 6.12 shows an internal or annular involute gear and pinion which has the advantage of reducing the required center distance for a given speed ratio as compared with a pair of spur gears. Internal gears will have greater length of contact, greater tooth strength and lower relative sliding between meshing teeth than external gears. In an internal gear the tooth profiles are concave instead of convex. The addendum of the annular is limited by the tangent point of the pinion's base circle and the pressure line, whereas the addendum of the pinion is unlimited except by the

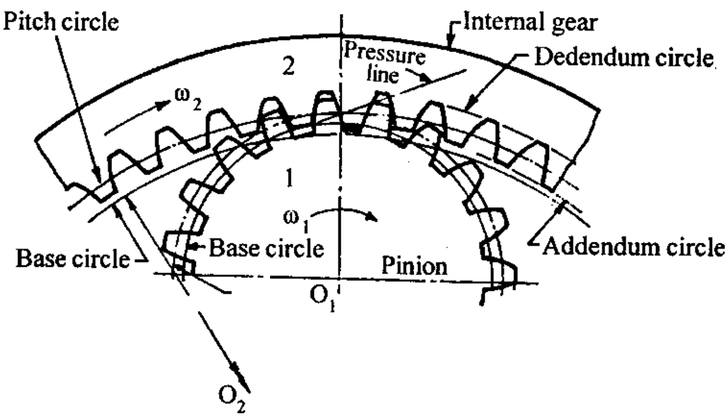

Fig. 6.12

teeth becoming pointed. If the ratio of the number of teeth in the pinion to the number of teeth in the annular exceed a certain limit, a type of interference known as *fouling* will occur between the teeth after they have ceased contact along the path of contact.

Example 6.1

A 3 mm module, 20° pinion, of 18 teeth drives a 45-tooth gear. Calculate the pitch radii, base radii, tooth thickness on the pitch circle.

Data:

$m = 3$ mm, $\phi = 20°$, $z_1 = 18$ teeth, $z_2 = 45$ teeth

Solution :

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{3 \times 18}{2} = 27$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{3 \times 45}{2} = 67.5$ mm

Base circle radius of pinion $r_{b_1} = r_1 \cos\phi$

$= 27 \cos 20 = 25.37$ mm

Base circle radius of gear $r_{b_2} = r_2 \cos\phi$

$= 67.5 \cos 20 = 63.43$ mm

Circular pitch $p = \pi m = \pi \times 3 = 9.425$ mm

Tooth thickness on pitch circle $t = \frac{p}{2} = \frac{9.425}{2} = 4.7125$ mm

---

Example 6.2

Determine the module of a pair of gears whose center distance is 58 mm. The gears have 18 and 40 teeth respectively. Also determine the pitch circle radii and circular pitch of the gears.

Data :

$c = 58$ mm, $z_1 = 18$ teeth, $z_2 = 40$ teeth

Solution :

Centre distance $c = r_1 + r_2 = \frac{mz_1}{2} + \frac{mz_2}{2} = \frac{m}{2}(z_1 + z_2)$ $\left[\because\ r = \frac{mz}{2}\right]$

i.e., $58 = \frac{m}{2} \times (18 + 40)$

$\therefore$ Module $m = 2$ mm

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{2 \times 18}{2} = 18$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{2 \times 40}{2} = 40$ mm

Circular pitch $p = \pi m = \pi \times 2 = 6.28$ mm

Example 6.3

A 20° involute spur gear running at 800 rpm drives another gear at a speed of 400 rpm. The center distance between the two gears is 600 mm and the module is 10 mm per tooth.

Determine the following:

1. Pitch circle radius of pinion
2. Pitch circle radius of gear
3. Number of teeth on pinion
4. Number of teeth on gear
5. Base circle radius of pinion
6. Base circle radius of gear
7. Circular pitch
8. Tooth thickness on pitch circle for no backlash

Data :

$\phi = 20°$, $n_1 = 800$ rpm, $n_2 = 400$ rpm, $c = 600$ mm, $m = 10$ mm

Solution :

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{r_2}{r_1} = \frac{z_2}{z_1}$$

i.e., $i = \frac{800}{400} = 2$

$\therefore$ $2 = \frac{r_2}{r_1}$

or $r_2 = 2\,r_1$

Center distance $c = r_l + r_2 = r_l + 2r_l = 3r_l$

i.e., $600 = 3\,r_l$

$\therefore$ Pitch circle radius of pinion $r_1 = 200$ mm

Pitch circle radius of gear $r_2 = 2\,r_1 = 2 \times 200 = 400$ mm

Number of teeth on pinion $z_1 = \frac{2r_1}{m} = \frac{2 \times 200}{10} = 40$ teeth

Number of teeth on gear $z_2 = i\,z_1 = 2 \times 40 = 80$ teeth

Base circle radius of pinion $r_{bl} = r_1 \cos\phi = 200 \times \cos 20 = 187.94$ mm

Base circle radius of gear $r_{b2} = r_2 \cos\phi = 400 \times \cos 20 = 375.88$ mm

Circular pitch $p = \pi m = \pi \times 10 = 31.42$ mm

Tooth thickness on pitch circle $t = \frac{p}{2} = \frac{31.42}{2} = 15.71$ mm

---

Example 6.4

Two gear wheels have respectively 28 and 45 teeth and a standard addendum of one module. The pressure angle is 20° and the module is 6mm. Find

a) Contact ratio (number of pairs in contact).

b) Angle turned by the pinion and the gear when one pair is in contact.

c) The ratio of sliding to rolling motion when the tip of a tooth on the larger wheel (i) is just making contact, (ii) is just leaving contact with its mating tooth, and (iii) is at the pitch point.

Data :

$z_1 = 28$ teeth $z_2 = 45$ teeth. $a_g = a_p = 1$, $\phi = 20°$, $m = 6$ mm

Solution :

Pitch circle radius of pinion $r_1 = \frac{m z_1}{2} = \frac{6 \times 28}{2} = 84$ mm

Pitch circle radius of gear $r_2 = \frac{m z_2}{2} = \frac{6 \times 45}{2} = 135$ mm

Addendum $a = a_1 = a_2 = a_g \times m = 1 \times 6 = 6$ mm

Addendum circle radius of pinion $r_{a_1} = r_1 + a_1 = 84 + 6 = 90$ mm

Addendum circle radius of gear $r_{a_2} = r_2 + a_2 = 135 + 6 = 141$ mm

Circular pitch $p = \pi m = 6\,\pi$ mm

Path of approach $A = \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} - r_2 \sin\phi$

$= \sqrt{141^2 - 135^2 \cos^2 20} - 135 \sin 20 = 15.37$ mm

Path of recess $R = \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - r_1 \sin\phi$

$= \sqrt{90^2 - 84^2 \cos^2 20} - 84 \sin 20 = 14.51$ mm

Path of contact = path of approach A + path of recess R

$= 15.37 + 14.51 = 29.88$ mm

(a) Contact raio (number of pairs of teeth in contact) $= \dfrac{\text{Path of contact}}{p\cos\phi}$

$$= \frac{29.88}{6\pi \times \cos 20} = 1.69$$

(b) Angle turned by pinion = Contact ratio × Angle subtended at the center by one tooth pitch

$$= 1.69 \times \frac{360}{28} = 21.73^\circ$$

Similarly, angle turned by gear $= 1.69 \times \dfrac{360}{45} = 13.52^\circ$

(c) Pitch line velocity of pinion $= \omega_1 r_1 = \omega_2 r_2$

$= \omega_1 \times 84 = 84\,\omega_1$

(i) When the tip of a large wheel is just making contact, the distance from the point of contact to the pitch point is the path of approach.

$\therefore$ Velocity of sliding $= (\omega_1 + \omega_2) \times$ path of approach

$$= \omega_1 \left(1 + \frac{\omega_2}{\omega_1}\right) \times \text{path of approach}$$

$$= \omega_1 \left(1 + \frac{z_2}{z_1}\right) \times \text{path of approach} \qquad \left\{\because \frac{\omega_2}{\omega_1} = \frac{z_1}{z_2}\right\}$$

$\therefore$ Velocity of sliding $= \omega_1 \left(1 + \dfrac{28}{45}\right) \times 15.37 = 24.93\,\omega_1$

$$\frac{\text{Velocity of sliding}}{\text{Pitch line velocity}} = \frac{24.93\,\omega_1}{84\,\omega_1} = 0.297$$

(ii) When the tip of larger wheel is just leaving contact, the distance from the point of contact to the pitch point is the path of recess.

$$\text{Velocity of sliding} = \omega_1\left(1+\frac{z_1}{z_2}\right)\times \text{path of recess}$$

$$= \omega_1\left(1+\frac{28}{45}\right)\times 14.51 = 23.54\ \omega_1$$

$$\frac{\text{Velocity of sliding}}{\text{Pitch line velocity}} = \frac{23.54\,\omega_1}{84\,\omega_1} = 0.28$$

(iii) At the pitch point there is no sliding.

$$\therefore \frac{\text{Velocity of sliding}}{\text{Pitch line velocity}} = \frac{0}{84\,\omega_1} = 0$$

Example 6.5

Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The gear has 57 teeth while the pinion has 23 teeth. If the addendum on pinion and gear are equal to one module, find (i) Number of pairs of teeth in contact, and (ii) Angle of action of pinion and gear

(VTU, July 2002)

Data:

$m = 8$ mm, $\phi = 20°$, $z_2 = 57$ teeth, $z_1 = 23$ teeth, $a_1 = a_2 = 1$ m $= 8$ mm

Solution:

Pitch circle radius of pinion $r_1 = \dfrac{mz_1}{2} = \dfrac{8\times 23}{2} = 92$ mm

Pitch circle radius of gear $r_2 = \dfrac{mz_2}{2} = \dfrac{8\times 57}{2} = 228$ mm

Addendum circle radius of pinion $r_{a_1} = r_1 + a_1 = 92 + 8 = 100$ mm

Addendum circle radius of gear $r_{a_2} = r_2 + a_2 = 228 + 8 = 236$ mm

Length of path of contact

$$= \sqrt{r_{a_2}^2 - r_2^2\cos^2\phi} + \sqrt{r_{a_1}^2 - r_1^2\cos^2\phi} - (r_1 + r_2)\sin\phi$$

$$= \sqrt{236^2 - 228^2\cos^2 20} + \sqrt{100^2 - 92^2\cos^2 20} - (92 + 228)\sin 20$$

$$= 39.7733 \text{ mm}$$

$$\text{Number of pairs of teeth in contact} = \frac{\text{Path of contact}}{p\cos\phi} = \frac{\text{Path of contact}}{\pi m\cos\phi}$$

$$= \frac{39.7733}{8\pi \times \cos 20} = 1.684$$

$$\text{Length of arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{39.7733}{\cos 20} = 42.326 \text{ mm}$$

$$\text{Angle of action of pinion} = \text{Arc of contact} \times \frac{360}{2\pi r_1}$$

$$= 42.326 \times \frac{360}{2\pi \times 92} = 26.36^\circ$$

$$\text{Angle of action of gear} = \text{Arc of contact} \times \frac{360}{2\pi r_2}$$

$$= 42.326 \times \frac{360}{2\pi \times 228} = 10.636^\circ$$

Example 6.6

Two spur wheels of equal diameter has 30 teeth each of involute shape. The circular pitch is 25 mm, and the pressure angle is 20°. Determine the addendum of the wheels, if the arc of contact is twice the circular pitch.

Data :

$z = z_1 = z_2 = 30$ teeth, $p = 25$ mm, $\phi = 20^\circ$, $d = d_1 = d_2$

Solution :

$$\text{Arc of contact} = 2 \times p$$

$$= 2 \times 25 = 50 \text{ mm}$$

$$\text{Path of contact} = \text{Arc of contact} \times \cos\phi$$

$$= 50 \times \cos 20 = 46.985 \text{ mm}$$

$$\text{Module } m = \frac{p}{\pi} = \frac{25}{\pi} \text{ mm}$$

$$\text{Pitch circle radius of gear } r = \frac{mz}{2} = \frac{25}{\pi} \times \frac{30}{2} = 119.37 \text{ mm}$$

$$\text{Path of contact} = \sqrt{r_{a2}^2 - r_2^2\cos^2\phi} + \sqrt{r_{a1}^2 - r_1^2\cos^2\phi} - (r_1 + r_2)\sin\phi$$

$$= 2\sqrt{r_a^2 - r^2\cos^2\phi} - 2r\sin\phi \quad (\because r_{a1} = r_{a2} = r_a \text{ and } r_1 = r_2 = r)$$

$$\text{i.e., } 46.985 = 2\sqrt{r_a^2 - 119.37^2\cos^2 20} - 2 \times 119.37 \sin 20$$

$\therefore$ Addendum circle radius $r_a = 129.3$ mm

$$\text{Addendum } a = r_a - r = 129.3 - 119.37 = 9.93 \text{ mm}$$

Example 6.7

The following data refer to two mating involute gear of 20° pressure angle. Number of teeth on pinion = 20, Gear ratio = 2, Speed of the pinion = 250 rpm, Module = 12 mm. If the addendum on each wheel is such that the path of approach and the path of recess on each side are half the maximum permissible length. Find the maximum velocity of sliding during approach and recess and the length of arc of contact. **(VTU, July 2003)**

Data:

$\phi = 20^\circ,\ z_1 = 20$ teeth, $i = 2,\ n_1 = 250$ rpm, $m = 12$ mm

Solution:

Number of teeth on gear $z_2 = i \times z_1 = 2 \times 20 = 40$

Pitch circle radius of pinion $r_1 = \dfrac{mz_1}{2} = \dfrac{12 \times 20}{2} = 120$ mm

Pitch circle radius of gear $r_2 = \dfrac{mz_2}{2} = \dfrac{12 \times 40}{2} = 240$ mm

Maximum permissible path of approach $= r_1 \sin\phi$

Required path of approach $= \dfrac{r_1 \sin\phi}{2} = \dfrac{120 \times \sin 20}{2} = 20.5212$ mm

Also path of approach $= \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} - r_2 \sin\phi$

i.e., $20.5212 = \sqrt{r_{a2}^2 - 240^2 \cos^2 20} - 240 \sin 20$

$\therefore$ Addendum circle radius of gear $r_{a_2} = 247.77$ mm

Addendum of gear $a_2 = r_{a_2} - r_2 = 247.77 - 240 = 7.77$ mm

Maximum permissible path of recess $= r_2 \sin\phi$

Required path of recess $= \dfrac{r_2 \sin\phi}{2} = \dfrac{240 \times \sin 20}{2} = 41.0424$ mm

Also, path of recess $= \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - r_1 \sin\phi$

i.e., $41.0424 = \sqrt{r_{a1}^2 - 120^2 \cos^2 20} - 120 \times \sin 20$

$\therefore$ Addendum circle radius of pinion $r_{a_1} = 139.476$ mm

Addendum of pinion $a_1 = r_{a_1} - r_1 = 139.476 - 120 = 19.476$ mm

Angular velocity of pinion $\omega_1 = \dfrac{2\pi n_1}{60} = \dfrac{2\pi \times 250}{60} = 26.18$ rad/s

Angular velocity of gear $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{\omega_1}{i} = \dfrac{26.18}{2} = 13.09$ rad/s

Maximum velocity of sliding during approach = $(\omega_1 + \omega_2) \times$ Path of approach

$= (26.18 + 13.09) \times 20.5212 = 805.868$ mm/s

Maximum velocity of sliding during recess = $(\omega_1 + \omega_2) \times$ Path of recess

$= (26.18 + 13.09) \times 41.0424 = 1611.735$ mm/s

Example 6.8

Two spur wheels of equal diameter has 48 teeth each of involute shape. The pitch circle radius is 96mm, and the addendum is 4.25 mm. Calculate the length of action and contact ratio if the pressure angle is 20°. **(VTU, Feb. 2003)**

Data :

$z = z_1 = z_2 = 48$ teeth, $r = r_1 = r_2 = 96$ mm, $a = 4.25$ mm, $\phi = 20°$

Solution :

Module $m = \dfrac{d}{z} = \dfrac{2r}{z} = \dfrac{2 \times 96}{48} = 4$ mm

Addendum circle radius $r_a = r + a = 96 + 4.25 = 100.25$ mm

$$\text{Path of contact} = \sqrt{r_{a2}^2 - r_2^2\cos^2\phi} + \sqrt{r_{a1}^2 - r_1^2\cos^2\phi} - (r_1 + r_2)\sin\phi$$

$$= 2\sqrt{r_a^2 - r^2\cos^2\phi} - 2r\sin\phi \quad (\because r_{a1} = r_{a2} = r_a \text{ and } r_1 = r_2 = r)$$

$$= 2\sqrt{100.25^2 - 96^2\cos^2 20} - 2 \times 96\sin 20 = 21.788 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Path of contact}}{p\cos\phi} = \frac{\text{Path of contact}}{\pi m\cos\phi} \qquad (\because p = \pi m)$$

$$= \frac{21.788}{\pi \times 4\cos 20} = 1.845$$

Example 6.9

Two equal 5 mm module, 20° involute gears mesh together such that the addendum circle of each gear passes through the interference point of the other. If the contact ratio is 1.622, calculate the number of teeth and the outside radius on each gear.

Data:

$d = d_1 = d_2$, $m = 5$ mm, $\phi = 20°$, contact ratio = 1.622

Solution:

$$\text{Contact ratio} = \frac{\text{Path of contact}}{\pi m\cos\phi}$$

i.e., $1.622 = \dfrac{\text{Path of contact}}{\pi \times 5 \times \cos 20}$

Length of path of contact = 23.94 mm

Maximum path of contact to avoid interference = $(r_1 + r_2)\sin\phi = 2r\sin\phi$ $(\therefore r_1 = r_2 = r)$

i.e., $23.94 = 2r\sin 20$

Pitch circle radius of the gear $r = 35$ mm

Number of teeth on gear $z = \dfrac{d}{m} = \dfrac{2r}{m} = \dfrac{2 \times 35}{5} = 14$

For equal diameter gears, path of contact = $2\sqrt{r_a^2 - r^2\cos^2\phi} - 2r\sin\phi$

i.e., $23.94 = 2\sqrt{r_a^2 - 35^2\cos^2 20} - 2 \times 35\sin 20$

$\therefore$ Addendum circle radius $r_a = 40.68$ mm

Example 6.10

Two spur gear wheels have 24 and 30 teeth and a standard addendum of 1 module. The pressure angle is 20°. Calculate the path of contact and arc of contact. **(VTU, March, 2001)**

Data :

$z_1 = 24$ teeth, $z_2 = 30$ teeth, $a = 1m$, $\phi = 20°$

Solution :

Pitch circle radius of pinion $r_1 = \dfrac{mz_1}{2} = \dfrac{m \times 24}{2} = 12\,m$ mm

Pitch circle radius of gear $r_2 = \dfrac{mz_2}{2} = \dfrac{m \times 30}{2} = 15\,m$ mm

Addendum circle radius of pinion $r_{a1} = r_1 + a_1 = 12\,m + m = 13m$ mm

Addendum circle radius of gear $r_{a2} = r_2 + a_2 = 15\,m + m = 16\,m$ mm

$$\text{Path of contact} = \sqrt{r_{a2}^2 - r_2^2\cos^2\phi} + \sqrt{r_{a1}^2 - r_1^2\cos^2\phi} - (r_1 + r_2)\sin\phi$$

$$= \sqrt{(16m)^2 - (15m)^2\cos^2 20} + \sqrt{(13m)^2 - (12m)^2\cos^2 20}$$

$$-(12m + 15m)\sin 20 = 4.805m \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{4.805m}{\cos 20} = 5.113m$$

Example 6.11

For two involute gears in mesh, with pinion as the driver, the arc of approach is not less than 1.1 times the circular pitch. If the pressure angle is 20° and the velocity ratio is 2.5, find

a) The least number of teeth on each gear, if interference is just avoided and

b) Addendum of the gear in terms of circular pitch.

Data :

$\phi = 20°$, $i = 2.5$, Arc of approach = 1.1 p

Solution :

Length of path of approach A = Arc of approach × cos ϕ

$$= 1.1\, p \times \cos 20 = 1.034\, p \qquad \text{...... (1)}$$

For interference is just avoided.

Path of approach $A = r_1 \sin \phi$

$$= r_1 \sin 20 \qquad \text{...... (2)}$$

Equating the equations (1) and (2) we get,

$$1.034\, p = r_1 \sin 20$$

or $$p = 0.3308\, r_1$$

Number of teeth on pinion $z_1 = \dfrac{\pi d}{p} = \dfrac{2\pi r_1}{p}$

$$= \frac{2\pi r_1}{0.3308 r_1} = 18.99 \approx 19 \text{ teeth}$$

Number of teeth on gear $z_2 = i \times z_1$

$$= 2.5 \times 19 = 47.5 \text{ teeth}$$

which is not admissible.

Therefore take teeth $z_1 = 20$

$$\therefore z_2 = i \times z_1 = 2.5 \times 20 = 50 \text{ teeth}$$

Pitch circle radius of the gear

$$r_2 = \frac{m z_2}{2} = \frac{p}{\pi} \times \frac{z_2}{2} \qquad \left(\because m = \frac{p}{\pi}\right)$$

$$= \frac{p \times 50}{2\pi} = 7.96\, p$$

Also, path of approach $A = \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi = 1.034\, p$

i.e., $\sqrt{r_{a2}^2 - (7.96\, p)^2 \cos^2 20} - 7.96\, p \times \sin 20 = 1.034\, p$

$\therefore$ Addendum circle radius of gear $r_{a_2} = 8.37\, p$ mm

Addendum $a_2 = r_{a_2} - r_2 = 8.37\, p - 7.96\, p = 0.41\, p$ mm

Example 6.12

Two spur gear of equal diameter, each with 25 teeth of involute form and a pressure angle of 20° are required to give an arc of contact equal to 1.6 times the circular pitch. Find the addendum required in terms of the circular pitch.

Data :

$d_1 = d_2 = d, z_1 = z_2 = z = 25$ teeth, $\phi = 20°$

Arc of contact = 1.6 p

Solution :

Pitch circle radius $r = \frac{mz}{2} = \frac{p}{\pi} \times \frac{25}{2} = 3.98\, p$ mm $\qquad \left(\because m = \frac{p}{\pi}\right)$

Length of path of contact = Arc of contact × cos ϕ

$= 1.6\, p \times \cos 20 = 1.5035\, p$ mm

Also, path of contact $= \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} + \sqrt{r_{a1}^2 - r_1^2 \cos^2 \phi} - (r_1 + r_2) \sin \phi$

$= 2\sqrt{r_a^2 - r^2 \cos^2 \phi} - 2r \sin \phi (\because r_{a_1} = r_{a_2} = r_a, r_1 = r_2 = r)$

i.e., $1.5035\, p = 2\sqrt{r_a^2 - (3.98p)^2 \cos^2 20} - 2 \times 3.98\, p \times \sin 20$

$\therefore$ Addendum circle radius $r_a = 4.3\, p$

Addendum $a = r_a - r = 4.3\, p - 3.98\, p = 0.32\, p$ mm

Example 6.13

For two involute gears in mesh, with pinion as the driver, the arc of approach is not less than 4 times the module. If the pressure angle is 20° and the velocity ratio is 2.5, find i) Least number of teeth on each gear if interference is just avoided and ii) Addendum on the gear interms of module. **(VTU, July-2004)**

Data:

Arc of approach = 4 m, $\phi = 20°$, $i = 2.5$

Solution:

Length of path of approach A = Arc of approach × cos ϕ

$= 4\, m \times \cos 20 = 3.7588\, m \qquad \text{..... (1)}$

For interference just avoided,

Path of approach $= r_1 \sin \phi = r_1 \sin 20 \qquad \text{..... (2)}$

Equating the equations (1) and (2), we get

$r_1 \sin 20 = 3.7588\, m$

$\therefore$ Pitch circle radius of pinion $r_1 = 10.9899\, m \qquad \text{..... (3)}$

Number of teeth on pinion $z_1 = \frac{d_1}{m} = \frac{2r_1}{m} = \frac{2 \times 10.9899m}{m} = 21.9798 \sim 22$

Number of teeth on gear $z_2 = i \times z_1 = 2.5 \times 22 = 55$

Pitch circle radius of gear $r_2 = \frac{d_2}{2} = \frac{mz_2}{2} = \frac{55m}{2} = 27.5\ m$

Also, path of approach $A = \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi = 3.7588\ m$

i.e., $\sqrt{r_{a2}^2 - (27.5m)^2 \cos^2 20} - 27.5\ m\ \sin 20 = 3.7588\ m$

$\therefore$ Addendum circle radius of gear $r_{a_2} = 29\ m$

Addendum $a_2 = r_{a_2} - r_2 = 29\ m - 27.5\ m = 1.5\ m$

Example 6.14

Two 20° involute spur gears have a module of 10 mm and addendum of one module. The number of teeth on pinion is 13 and on the gear is 52. Does interference occur? If it occurs, to what value should the pressure angle be changed to eliminate the interference?

Data :

$\phi = 20°$, $m = 10$ mm, $a = 1\ m$, $z_1 = 13$ teeth, $z_2 = 52$ teeth

Solution :

Pitch circle radius of pinion $r_1 = \frac{m z_1}{2} = \frac{10 \times 13}{2} = 65$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{10 \times 52}{2} = 260$ mm

Addendum $a = 1m = 1 \times 10 = 10$ mm

Addendum circle radius of gear $r_{a_2} = r_2 + a = 260 + 10 = 270$ mm

Maximum addendum circle radius of the gear to avoid interferences is

$$(r_{a2})_{\max} = \sqrt{r_2^2 + r_1^2 \sin^2 \phi + 2r_1 r_2 \sin^2 \phi}$$

$$= \sqrt{260^2 + 65^2 \sin^2 20 + 2 \times 65 \times 260 \times \sin^2 20}$$

$$= 268.42 \text{ mm}$$

Since the actual value of r_{a_2} is greater than $(r_{a_2})_{max}$, there will be interference. The pressure angle to avoid interference is

$$270 = \sqrt{260^2 + 65^2 \sin^2 \phi + 2 \times 65 \times 260 \times \sin^2 \phi}$$

Squaring both sides we get,

$$270^2 = 260^2 + \sin^2\phi(65^2 + 2 \times 65 \times 260)$$

$$\therefore \text{Pressure angle } \phi = 21.92^\circ$$

Example 6.15

A 14½ ° involute pinion with 20 teeth drives a 50 teeth gear. The pitch circle diameter of the pinion is 160 mm. Find the addenda of the pinion and gear, and the contact ratio without interference.

Data :

$\phi = 14½°$, $z_1 = 20$ teeth, $z_2 = 50$ teeth, $d_1 = 160$ mm

Solution :

Pitch circle diameter of pinion $d_1 = mz_1$

i.e., $160 = m \times 20$

$\therefore$ Module $m = 8$ mm

Pitch circle radius of pinion $r_1 = \dfrac{d_1}{2} = \dfrac{160}{2} = 80$ mm

Pitch circle radius of gear $r_2 = \dfrac{mz_2}{2} = \dfrac{8 \times 50}{2} = 200$ mm

Maximum addendum circle radius of gear to avoid interference is,

$$r_{a_2} = \sqrt{r_2^2 + r_1^2 \sin^2\phi + 2r_1 r_2 \sin^2\phi}$$

$$= \sqrt{200^2 + 80^2 \sin^2 14.5 + 2 \times 80 \times 200 \sin^2 14.5}$$

$$= 205.93 \text{ mm}$$

$\therefore$ Addendum of gear $a_2 = r_{a2} - r_2 = 205.93 - 200 = 5.93$ mm

Similarly, the maximum addendum circle radius of pinion to avoid interference is.

$$r_{a_1} = \sqrt{r_1^2 + r_2^2 \sin^2\phi + 2r_1 r_2 \sin^2\phi}$$

$$= \sqrt{80^2 + 200^2 \sin^2 14.5 + 2 \times 80 \times 200 \sin^2 14.5}$$

$$= 104.47 \text{ mm}$$

$\therefore$ Addendum of pinion $a_1 = r_{a_1} - r_1 = 104.47 - 80 = 24.47$ mm

Maximum path of contact to avoid interference is

$$= (r_1 + r_2)\sin\phi = (80 + 200)\sin 14.5 = 70.106 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Path of contact}}{p\cos\phi} = \frac{\text{Path of contact}}{\pi\, m\cos\phi}$$

$$= \frac{70.106}{\pi \times 8 \times \cos 14.5} = 2.88$$

Example 6.16

The following data refers to the two mating involute gears of 20° pressure angle.

Number of teeth on pinion = 30

Gear ratio = 2

Module = 12 mm

Speed of pinion = 600 rpm

The line of contact on each side of the pitch point is half the maximum possible length. Find the height of addendum for each gear wheel and the length of arc of contact. Also find the maximum velocity of sliding during approach and recess.

Data :

$\phi = 20^\circ$, $z_1 = 30$ teeth, $i = 2$, $m = 12$ mm, $n_1 = 600$

Solution :

Number of teeth on gear $z_2 = i \times z_1 = 2 \times 30 = 60$ teeth

Pitch circle radius of pinion $r_1 = \frac{m z_1}{2} = \frac{12 \times 30}{2} = 180$ mm

Pitch circle radius of gear $r_2 = \frac{m z_2}{2} = \frac{12 \times 60}{2} = 360$ mm

When the interference is just avoided, contact of tooth begins at C and ends at points D as shown in fig. 6.13 (Pinion is the driver).

Maximum path of approach $A = \text{CP} = r_1 \sin\phi$

Maximum path recess $R = \text{PD} = r_2 \sin\phi$

By data, the required path of approach $A = \frac{r_1 \sin\phi}{2}$

$$= \frac{180 \times \sin 20}{2} = 30.78 \text{ mm} \qquad \text{......(1)}$$

and required path of recess $= \frac{r_2 \sin\phi}{2}$

$$= \frac{360 \sin 20}{2} = 61.56 \text{ mm} \qquad \text{......(2)}$$

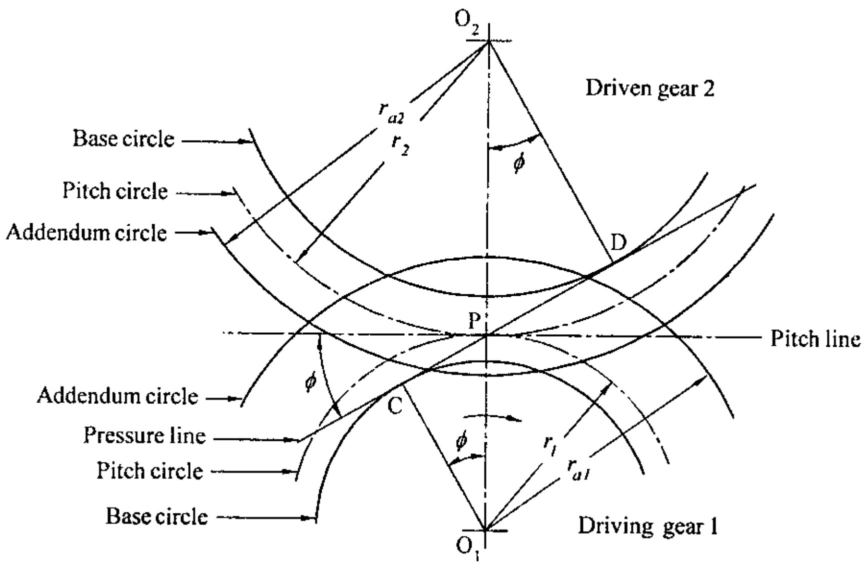

Fig. 6.13

Also, the path of approach $= \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi$ ...... (3)

and the path of recess $= \sqrt{r_{a1}^2 - r_1^2 \cos^2 \phi} - r_1 \sin \phi$ ...... (4)

Equating (1) and (3) we get,

$$\sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi = 30.78$$

i.e., $\sqrt{r_{a2}^2 - 360^2 \cos^2 20} - 360 \sin 20 = 30.78$

$$\sqrt{r_{a2}^2 - 114439.68} = 153.9$$

Squaring both sides, we get

$$r_{a2}^2 - 114439.68 = 23685.2$$

$\therefore$ Addendum circle radius of gear $r_{a_2} = 371.65$ mm

Addendum of gear $a_2 = r_{a_2} - r_2$

$= 371.65 - 360 = 11.65$ mm

Similarly equating (2) and (4) we get,

$$\sqrt{r_{a1}^2 - 180^2 \cos^2 20} - 180 \sin 20 = 61.56$$

$\therefore$ Addendum circle radius of pinion $r_{a1} = 209.21$ mm

Addendum of pinion $a_1 = r_{a_1} - r_1 = 209.21 - 180 = 29.21$ mm

$$\text{Path of contact} = \text{Path of approach } A + \text{Path of recess } R$$
$$= 30.78 + 61.56 = 92.34 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{92.34}{\cos 20} = 98.27 \text{ mm}$$

$$\text{Angular velocity of pinion } \omega_1 = \frac{2\pi n}{60} = \frac{2\pi \times 600}{60} = 62.8 \text{ rad / s}$$

$$\text{Angular velocity of gear } \omega_2 = \frac{\omega_1}{i} = \frac{62.8}{2} = 31.4 \text{ rad/s}$$

Maximum velocity of sliding during approach

$$= (\omega_1 + \omega_2) \times \text{path of approach}$$
$$= (62.8 + 31.4) \times 30.78 = 2899.5 \text{ mm / s}$$

Maximum velocity of sliding during recess $= (\omega_1 + \omega_2) \times$ path of recess

$$= (6.28 + 31.4) \times 61.5 = 5799 \text{ mm/s}$$

Example 6.17

The following data relate to two meshing involute gears. Number of teeth on gear = 60, Pressure angle = 20°, Gear ratio = 1.5, Speed of the driver (gear) = 100 rpm, Module = 8 mm. The addendum on each wheel is such that the path of approach and the path of recess on each side are 40% of the maximum possible length each. Determine the addendum for the pinion and gear and the length of arc of contact.

Data:

$z_2 = 60$ teeth, $\phi = 120°$, $i = 1.5$, $n_2 = 100$ rpm, $m = 8$ mm

Path of approach = 40% maximum possible path of approach = $0.4 \times r_2 \sin\phi$

Path of recess = 40% of maximum possible path of recess = $0.4 \times r_1 \sin\phi$

Solution:

$$\text{Number of teeth on pinion } z_1 = \frac{z_2}{i} = \frac{60}{1.5} = 40$$

$$\text{Pitch circle radius of pinion } r_1 = \frac{mz_1}{2} = \frac{8 \times 40}{2} = 160 \text{ mm}$$

$$\text{Pitch circle radius of gear } r_2 = \frac{mz_2}{2} = \frac{8 \times 60}{2} = 240 \text{ mm}$$

Required path of approach = $0.4 \times r_2 \sin\phi$ (∵ Gear is the driver)

$$= 0.4 \times 240 \sin 20 = 32.834 \text{ mm}$$

Also, path of approach = $\sqrt{r_{a1}^2 - r_1^2 \cos^2 \phi} - r_1 \sin \phi$

i.e., $\sqrt{r_{a1}^2 - 160^2 \cos^2 20} - 160 \sin 20 = 32.834$

$\therefore$ Addendum circle radius of pinion $r_{a1} = 173.99$ mm

Addendum of pinion $a_1 = r_{a1} - r_1 = 173.99 - 160 = 13.99$ mm

Similarly, required path of recess = $0.4 \times r_1 \sin \phi$ ($\because$ Gear is the driver)

$= 0.4 \times 160 \sin 20 = 21.889$ mm

Path of recess = $\sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi$

i.e., $\sqrt{r_{a2}^2 - 240^2 \cos^2 20} - 240 \sin 20 = 21.889$

$\therefore$ Addendum circle radius of gear $r_{a2} = 248.34$ mm

Addendum of gear $a_2 = r_{a2} - r_2 = 248.34 - 240 = 8.34$ mm

Path of contact = path of apprach + path of recess

$= 32.834 + 21.889 = 54.723$ mm

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{54.723}{\cos 20} = 58.235 \text{ mm}$$

Example 6.18

A pair of gears have 14 and 16 teeth and the module is 12.5 mm. The addendum is 12.5 mm and the pressure angle 14.5°. Show that the gears have interference. Determine the addenda of gears to avoid interference. Find the length of path of contact for the new addenda.

Data:

$z_1 = 14$ teeth, $z_2 = 16$ teeth, $m = 12.5$ mm, $a_1 = a_2 = 12.5$ mm, $\phi = 14.5°$

Solution:

$$\text{Pitch circle radius of pinion } r_1 = \frac{mz_1}{2} = \frac{12.5 \times 14}{2} = 87.5 \text{ mm}$$

$$\text{Pitch circle radius of gear } r_2 = \frac{mz_2}{2} = \frac{12.5 \times 16}{2} = 100 \text{ mm}$$

Addendum circle radius of pinion $r_{a1} = r_1 + a_1 = 87.5 + 12.5 = 100$ mm

Addendum circle radius of gear $r_{a2} = r_2 + a_2 = 100 + 12.5 = 112.5$ mm

Actual path of approach $= \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} - r_2 \sin \phi$

$= \sqrt{112.5^2 - 100^2 \cos^2 14.5} - 100 \sin 14.5 = 32.26$ mm

Maximum path of approach $= r_1 \sin\phi = 87.5 \sin 14.5 = 21.908$ mm

As the actual path of approach is greater than the maximum permissible value, interference will exist. To eliminate interference, equate the actual path of approach to its maximum permissible value.

i.e., $\sqrt{r_{a2}^2 - 100^2 \cos^2 14.5} - 100 \sin 14.5 = 21.908$

$\therefore$ New addendum circle radius of gear $ra_2 = 107.597$ mm

New addendum for gear, $a_2 = r_{a2} - r_2$

$= 107.597 - 100 = 7.597$ mm

Similarly, maximum path of recess $= r_2 \sin\phi$

$= 100 \sin 14.5 = 25.038$ mm

i.e., $\sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - r_1 \sin\phi = 25.038$

$\sqrt{r_{a1}^2 - 87.5^2 \cos^2 14.5} - 87.5 \sin 14.5 = 25.038$

$\therefore$ New addendum circle radius of pinion $r_{a1} = 96.852$ mm

New addendum for pinion $a_1 = r_{a1} - r_1 = 96.852 - 87.5 = 9.352$ mm

Path of contact = Path of approach + Path of recess

$= 21.908 + 25.038 = 46.946$ mm

Example 6.19

Two mating gears have 50 and 13 involute teeth of module 10 mm and 20° pressure angle. The addendum is one module. Does the interference occur? If it occurs, to what value should be the pressure angle be changed to eliminate interference.

Data:

$z_1 = 13$ teeth, $z_2 = 50$ teeth, $m = 10$ mm, $\phi = 20°$, $a = 1\,m = 10$ mm

Solution:

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{10 \times 13}{2} = 65$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{10 \times 50}{2} = 250$ mm

Addendum circle radius of pinion $r_{a1} = r_1 + a = 65 + 10 = 75$ mm

Addendum circle radius of gear $r_{a2} = r_2 + a = 250 + 10 = 260$ mm

Actual path of approach $= \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} - r_2 \sin\phi$

$= \sqrt{260^2 - 250^2 \cos^2 20} - 250 \sin 20 = 25.9$ mm

Maximum path of approach $= r_1 \sin\phi$

$$= 65 \sin 20 = 22.23 \text{ mm}$$

As the actual length of approch is more than the maximum permissible value, hence interference will occur.

To eliminate interference, equate the actual path of approach to maximum path of approach, we get,

$$65 \sin\phi_1 = 25.9$$

$\therefore$ Pressure angle to eliminate interference $\phi_1 = 23.48°$

Example 6.20

A pinion with 30 involute teeth of 4 mm module, gears with a rack. If the pressure angle is 20°, and the addendum for pinion and the rack are same, determine

i) Maximum addendum if the interference is to be avoided

ii) The length of the resulting path of contact.

Data :

$z = 30$ teeth, $m = 4$mm, $\phi = 20°$

Solution :

Pitch circle radius of pinion $r = \frac{mz}{2} = \frac{4 \times 30}{2} = 60$ mm

Maximum addendum of the rack $a = r\sin^2\phi = 60 \sin^2 20 = 7.02$ mm

Addendum circle radius of pinion $r_a = r + a = 60 + 7.02 = 67.02$ mm

$$\text{Path of contact} = \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi + \frac{a}{\sin\phi}$$

$$= \sqrt{67.02^2 - 60^2\cos^2 20} - 60\sin 20 + \frac{7.02}{\sin 20} = 36.236 \text{ mm}$$

Example 6.21

A straight tooth pinion with teeth of 6 mm module, and 20° pressure angle meshes with a rack. If the addendum of the pinion and the rack is 6 mm, find the maximum number of teeth on pinion for interference to be avoided. If the pinion had 3 teeth less than this number, how much addendum correction would be required?

Data :

$m = 6$ mm, $\phi = 20°$, $a = 6$ mm

Solution :

Addendum of the rack $a = a_r\, m$

i.e., $6 = a_r \times 6$

$\therefore$ Addendum coefficient of the rack $a_r = 1$

Minimum number of teeth on pinion to avoid inteference is

$$z = \frac{2a_r}{\sin^2\phi} = \frac{2\times 1}{\sin^2 20} = 17.09 \simeq 18 \text{ teeth}$$

Number of teeth on pinion after reducing 3 teeth is,

$$z = 18 - 3 = 15$$

$$\text{Also, } z = \frac{2a_r}{\sin^2\phi}$$

$$\text{i.e, } 15 = \frac{2a_r}{\sin^2 20}$$

$\therefore$ Addendum coefficient of the rack $a_r = 0.877$

New value of the addendum $a = a_r\, m = 0.877 \times 6 = 5.264$ mm

Addendum correction $= 6 - 5.264 = 0.736$ mm

Example 6.22

What is the smallest number of teeth theoretically required in order to avoid interference on a pinion which gear with a) rack, b) an equal diameter gear, and c) a wheel to give a gear ratio of 3 :1. The addendum is one module and the pressure angle is 20°.

Data :

$i = 3,\ \phi = 20°,\ a_r = a_p = a_g = 1$

Solution :

a) For rack and pinion :

Number of teeth on pinion $z = \dfrac{2a_r}{\sin^2\phi} = \dfrac{2\times 1}{\sin^2 20} = 17.09 \approx 18$ teeth

b) For the gear ratio of unity :

Number of teeth $z_1 = z_2 = \dfrac{2a_g}{\sqrt{1+3\sin^2\phi}-1} = \dfrac{2\times 1}{\sqrt{1+3\sin^2 20}-1} = 12.3 \approx 13$ teeth *c) For gear ratio of 3 : 1*

$$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1} = \frac{2\times 1}{\sqrt{1+\frac{\sin^2 20}{3}\left(\frac{1}{3}+2\right)}-1} = 44.94 \approx 45 \text{ teeth}$$

Number of teeth on pinion $z_1 = \frac{z_2}{i} = \frac{45}{3} = 15$ teeth

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Example 6.23

Two gear wheels mesh externally are to give a velocity ratio of 3. The teeth are of involute form of module 6 mm. and the pressure angle is 20°. The standard addendum is one module and the pinion rotates at 400 rpm. Find

i) The number of teeth on each wheel, so that the interferences is just avoided.
ii) Length of path of contact.
iii) The maximum velocity of sliding between the teeth.
iv) Arc of contact, and
v) Number of pairs of teeth in contact. **(VTU, Aug, 2005)**

Data :

$i = 3$, $m = 6$ mm, $\phi = 20°$, $a_g = 1$, $n_1 = 400$ rpm

Solution :

The minimum number of teeth on the gear to avoid interference is

$$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1} = \frac{2\times 1}{\sqrt{1+\frac{\sin^2 20}{3}\left(\frac{1}{3}+2\right)}-1} = 44.94$$

Take the number of teeth on gear $z_2 = 45$ teeth (divisible by i)

Number of teeth on pinion $z_1 = \frac{z_2}{i} = \frac{45}{3} = 15$ teeth

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{6\times 15}{2} = 45$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{6\times 45}{2} = 135$ mm

Addendum $a = a_1 = a_2 = a_g\, m = 1\times 6 = 6$ mm

Addendum circle radius of gear $r_{a_2} = r_2 + a_2 = 135 + 6 = 141$ mm

Addendum circle radius of pinion $r_{a_1} = r_1 + a_1 = 45 + 6 = 51$ mm

Path of approach $A = \sqrt{r_{a2}^2 - r_2^2 \cos^2\phi} - r_2 \sin\phi$

$= \sqrt{141^2 - 135^2 \cos^2 20} - 135 \sin 20 = 15.37$ mm

Path of recess $R = \sqrt{r_{a1}^2 - r_1^2 \cos^2\phi} - r_1 \sin\phi$

$= \sqrt{51^2 - 45^2 \cos^2 20} - 45 \sin 20 = 13.12$ mm

Length of path of contact = Path of approach A + Path of recess R

$= 15.37 + 13.12 = 28.49$ mm

Angular velocity of pinion $\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 400}{60} = 41.9$ rad / s

Angular velocity of gear $\omega_2 = \frac{\omega_1}{i} = \frac{41.9}{3} = 13.97$ rad / s

The maximum velocity of sliding occurs during approach.

$\therefore$ Maximum velocity of sliding $= (\omega_1 + \omega_2) \times$ path of approach

$= (41.9 + 13.97) \times 15.37 = 858.7$ mm/s

Arc of contact $= \frac{\text{path of contact}}{\cos\phi} = \frac{28.49}{\cos 20} = 30.32$ mm

Circular pitch $p = \pi\ m = 6\ \pi$ mm

Number of pairs of teeth in contact $= \frac{\text{path of contact}}{p \cos\phi} = \frac{28.49}{6\pi \times \cos 20} = 1.608$

Example 6.24

A 20 teeth pinion of involute tooth profile has circular pitch equal to 12.6 mm. The gear ratio is 2 and the addendum is 4 mm. Determine the least pressure angle which may be used to avoid undercutting.

Data :

$z_1 = 20$ teeth, $p = 12.6$ mm, $i = 2$, $a_1 = a_2 = a = 4$ mm

Solution :

Module $m = \frac{p}{\pi} = \frac{12.6}{\pi} = 4$ mm

Addendum $a_2 = a_g\ m$

i.e., $4 = a_g \times 4$

$\therefore$ $a_g = 1$

The number of teeth on gear $z_2 = i\,z_1 = 2 \times 20 = 40$ teeth

Also $$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1}$$

i.e., $$40 = \frac{2 \times 1}{\sqrt{1+\frac{\sin^2\phi}{3}\left(\frac{1}{2}+2\right)}-1}$$

or $$\sqrt{1+1.25\sin^2\phi} = 1.05$$

Squaring both sides we get,

$$1 + 1.25\sin^2\phi = 1.1025$$

$\therefore$ Pressure angle $\phi = 16.64°$

Example 6.25

A pair of spur gears has 16 teeth and 18 teeth, a module 12.5mm, an addendum 12.5mm and a pressure angle 14.5°. Prove that the gears have interference. Determine the minimum number of teeth and the velocity ratio to avoid interference. **(VTU, Jan. 2005)**

Data :

$z_1 = 16$ teeth, $z_2 = 18$ teeth, $m = 12.5$ mm, $a = 12.5$ mm, $\phi = 14.5°$

Solution :

Addendum $a_2 = a_g\, m$

i.e., $12.5 = a_g \times 12.5$

$\therefore$ Addendum coefficient $a_g = 1$

Gear ratio $$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{18}{16} = 1.125$$

Minimum number of teeth on gear to avoid interference is,

$$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1} = \frac{2 \times 1}{\sqrt{1+\frac{\sin^2 14.5}{1.125}\left(\frac{1}{1.125}+2\right)}-1} = 25.8$$

The given number of teeth on gear is less than the minimum number of teeth required to avoid interference. Therefore interference exist. Select the value z_2 greaterthan 25.8.

Take $z_2 = 27$, (divisible by i)

$\therefore$ Number of teeth on pinion $$z_1 = \frac{z_2}{i} = \frac{27}{1.125} = 24$$

3

Example 6.26

A pair of spur wheels with involute teeth is to give a gear ratio 3 : 1. The arc of approach is not less than the circular pitch and the smaller wheel is the driver. The pressure angle is 20°. What is the least number of teeth that can be used on each wheel? What is the addendum of wheel interms of circular pitch?

Data:

$i = 3$, arc of approach $\geq p$, $\phi = 20°$

Solution:

Maximum path of approach $= r_1 \sin\phi$ (∵ Pinion is the driver)

$$\text{Arc of approach} = \frac{\text{Path of approach}}{\cos\phi} = \frac{r_1 \sin\phi}{\cos\phi} = r_1 \tan\phi$$

By data, arc of approach $\geq p$ (∵ $p = \pi m$)

i.e., $r_1 \tan\phi \geq \pi m$

or $\frac{m z_1}{2} \tan\phi \geq \pi m$ (∵ $d_1 = m z_1$)

$$\frac{z_1}{2} \times \tan 20 \geq \pi$$

∴ Number of teeth on pinion $z_1 \geq 17.26$

Take $z_1 = 18$

∴ Number of teeth on gear $z_2 = i \times z_1 = 3 \times 18 = 54$ teeth

Minimum number of teeth on gear to avoid interference is

$$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1}$$

i.e.,

$$54 = \frac{2a_g}{\sqrt{1+\frac{\sin^2 20}{3}\left(\frac{1}{3}+2\right)}-1}$$

Addendum coefficient $a_g = 1.20153$

Addendum of gear $a_2 = a_g m = \frac{a_g p}{\pi}$ (∵ $p = \pi m$)

$$= \frac{1.20153\,p}{\pi} = 0.382\,p \text{ mm}$$

Example 6.27

Two 20° involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 and the addendum is equal to 1.1 module. If the pinion rotates at 120 rpm, determine: (i) Minimum number of teeth on each wheel to avoid interference (ii) Number of pairs of teeth in contact.

(VTU, Jan. 2007)

Data:

$\phi = 20°,\ i = 3,\ m = 3 \text{ mm},\ a_g = 1.1\ m,\ n_1 = 120 \text{ rpm}$

Solution:

Minimum number of teeth on gear to avoid interference is

$$z_2 = \frac{2a_g}{\sqrt{1+\frac{\sin^2\phi}{i}\left(\frac{1}{i}+2\right)}-1}$$

$$= \frac{2\times 1.1}{\sqrt{1+\frac{\sin^2 20}{3}\left(\frac{1}{3}+2\right)}-1} = 49.43$$

Take the number of teeth on gear $z_2 = 51$ (divisible by i)

Number of teeth on pinion $z_1 = \frac{z_2}{i} = \frac{51}{3} = 17$

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{3\times 17}{2} = 25.5$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{3\times 51}{2} = 76.5$ mm

Addendum $a = a_1 = a_2 = a_g\, m = 1.1 \times 3 = 3.3$ mm

Addendum circle radius of pinion $r_{a1} = r_1 + a_1 = 25.5 + 3.3 = 28.8$ mm

Addendum circle radius of gear $r_{a2} = r_2 + a_2 = 76.5 + 3.3 = 79.8$ mm

Length of path of contact $= \sqrt{r_{a2}^2 - r_2^2\cos^2\phi} + \sqrt{r_{a1}^2 - r_1^2\cos^2\phi} - (r_1 + r_2)\sin\phi$

$= \sqrt{79.8^2 - 76.5^2\cos^2 20} + \sqrt{28.8^2 - 25.5^2\cos^2 20} - (25.5 + 76.5)\sin 20$

$= 15.737$

Number of pair of teeth in contact $= \frac{\text{Path of contact}}{p\cos\phi} = \frac{15.737}{\pi\times 3\times\cos 20} = 1.7769$

Example 6.28

Two gear wheels mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6 mm and standard addendum of one module pressure angle is 18°. The pinion rotates at 90 rpm. Find, (i) Number of teeth on each wheel so that interference is just avoided. (ii) Length of path of contact. (iii) Maximum velocity of sliding between teeth.

(VTU, Feb 2004)

Data:

$i = 3,\ m = 6\text{ mm},\ a = a_1 = a_2 = 1\ m = 1 \times 6 = 6\text{ mm},\ a_g = 1,\ \phi = 18°,\ n_1 = 90\text{ rpm}$

Solution:

Minimum number of teeth on gear to avoid interference is

$$z_2 = \frac{2a_g}{\sqrt{1 + \frac{\sin^2\phi}{i}\left(\frac{1}{i} + 2\right)} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{\sin^2 18}{3}\left(\frac{1}{3} + 2\right)} - 1} = 54.84$$

Take the number of teeth on gear $z_2 = 57$ (divisible by i)

$\therefore$ Number of teeth on pinion $z_1 = \frac{z_2}{i} = \frac{57}{3} = 19$

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{6 \times 19}{2} = 57\text{ mm}$

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{6 \times 57}{2} = 171\text{ mm}$

Addendum circle radius of pinion $r_{a1} = r_1 + a = 57 + 6 = 63\text{ mm}$

Addendum circle radius of gear $r_{a2} = r_2 + a = 171 + 6 = 177\text{ mm}$

Length of path of approach $= \sqrt{r_{a2}^2 - r_2^2\cos^2\phi} - r_2\sin\phi$

$$= \sqrt{177^2 - 171^2\cos^2 18} - 171\sin 18 = 17.017\text{ mm}$$

Length of path of recess $= \sqrt{r_{a1}^2 - r_1^2\cos^2\phi} - r_1\sin\phi$

$$= \sqrt{63^2 - 57^2\cos^2 18} - 57\sin 18 = 14.484\text{ mm}$$

Angular velocity of pinion $\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 90}{60} = 9.425\text{ mm/s}$

Angular velocity of gear $\omega_2 = \frac{\omega_1}{i} = \frac{9.425}{3} = 3.1417\text{ mm/s}$

The maximum velocity of sliding occurs during approach.

Maximum velocity of sliding during approach = $(\omega_1 + \omega_2) \times$ length of approach

$$= (9.425 + 3.1417) \times 17.017 = 213.85 \text{ mm/s}$$

Example 6.29

The following are the particulars of a single reduction spur gear. The gear ratio is 10 to 1 and the center distance is 275 mm. The pinion transmits 3.75 kW at 1800 rpm. The teeth are of involute form with standard addendum of one module and pressure angle is 22.5 degrees. The normal tooth pressure not to exceed 9810 N/cm width. Find:

i) The nearest standard module if no interference is to occur.

ii) The number of teeth in each wheel. **(VTU, Feb, 2002)**

Data :

$i = 10$, $c = 275$ mm, $P = 3.75$ kW, $n_1 = 1800$ rpm, $a = 1\,m$, $a_g = 1$, $\phi = 22.5°$

Solution :

Minimum number of teeth on gear to avoid interference is

$$z_2 = \frac{2\,a_g}{\sqrt{1 + \frac{\sin^2\phi}{i}\left(\frac{1}{i} + 2\right)} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{\sin^2 22.5}{10}\left(\frac{1}{10} + 2\right)} - 1} = 131.05$$

Take number of teeth on gear as $z_2 = 140$ (divisible by i)

$\therefore$ Number of teeth on pinion $z_1 = \frac{140}{10} = 14$ teeth

Center distance $c = \frac{d_1 + d_2}{2} = \frac{m(z_1 + z_2)}{2}$

i.e., $275 = \frac{m(14 + 140)}{2}$

$\therefore$ Module $m = 3.57$

Take a standard module $m = 4$ mm

Pitch circle diameter of pinion $d_1 = m\,z_1 = 4 \times 14 = 56$ mm

Pitch circle diameter of gear $d_2 = m\,z_2 = 4 \times 140 = 560$ mm

$\therefore$ Correct center distance $c = \frac{(56 + 560)}{2} = 308$ mm

Involutometry

The study of the geometry of the involute is called *involutometry*. Consider an involute which has been generated from a base circle of radius r_B and C and D are two points on the involute as shown in fig. 6.14. Draw normal to the involute from points C and D. The normals CF and DG are tangent to base circle, and point B is the origin of the involute.

Let r_C = Radius of point C on the involute

r_D = Radius of point D on the involute

ϕ_C = Pressure angle at the point C

ϕ_D = Pressure angle at the point D

t_C = Arc tooth thickness at C

t_D = Arc tooth thickness at D

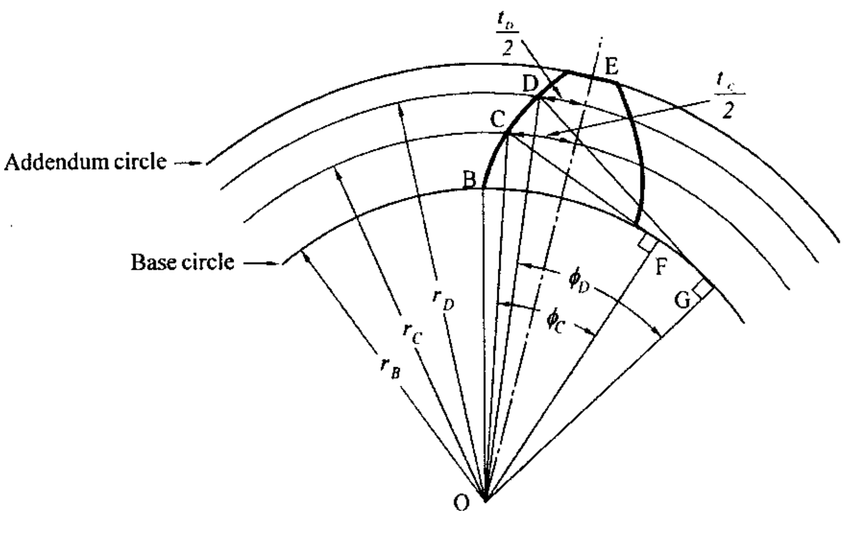

Fig. 6.14

From triangle O C F,

$$\cos \phi_C = \frac{OF}{OC} = \frac{r_B}{r_C}$$

$\therefore$ Base circle radius $r_B = r_C \cos \phi_C$ ...... (1)

From triangle O D G,

$$\cos \phi_D = \frac{OG}{OD} = \frac{r_B}{r_D}$$

$\therefore$ Base circle radius $r_B = r_D \cos \phi_D$ ...... (2)

From equations (1) and (2) we get

$$r_C \cos \phi_C = r_D \cos \phi_D \quad \text{...... (3)}$$

By the properties of involute,

$$\text{arc length BF} = \text{length of generating ling CF}$$
$$\text{arc length BG} = \text{length of generating line DG}$$

From the figure,

$$\angle BOF = \frac{\text{arc BF}}{OF} = \frac{CF}{OF} = \tan \phi_C$$

The angle between the radius vector defining the origin of the involute B and the point C on the involute is

$$\angle BOC = \angle BOF - \phi_C$$
$$\text{i.e., inv } \phi_C = \tan \phi_C - \phi_C \quad \text{......(4)}$$

The expression $\tan \phi - \phi$ is called an *involute function* inv ϕ and the angle ϕ is expressed in radians.

Similarly for point D, from the figure

$$\angle BOG = \frac{\text{arc BG}}{OG} = \frac{DG}{OG} = \tan \phi_D$$
$$\text{and } \angle BOD = \angle BOG - \phi_D$$
$$\therefore \text{inv } \phi_D = \tan \phi_D - \phi_D \quad \text{......(5)}$$

From the figure, for point C,

$$\angle BOE = \angle BOC + \frac{t_c}{2r_c} = \text{inv } \phi_C + \frac{t_c}{2r_c} \quad \text{......(6)}$$

From the figure, for point D,

$$\angle BOE = \angle BOD + \frac{t_D}{2r_D} = \text{inv } \phi_D + \frac{t_D}{2r_D} \quad \text{......(7)}$$

Equating the equations (6) and (7), we get

$$\text{inv } \phi_C + \frac{t_C}{2r_C} = \text{inv } \phi_D + \frac{t_D}{2r_D}$$

$$\therefore \text{Tooth thickness at D, } t_D = 2\,r_D\left[\frac{t_C}{2r_C} + \text{inv } \phi_C - \text{inv } \phi_D\right] \quad \text{..... (8)}$$

From the above equation, it is possible to calculate the tooth thickness at any point on the involute, if the tooth thickness at some other point is known.

Example 6.30

The thickness of an involute gear tooth is 7.98 mm at a radius of 88.9 mm and pressure angle of 14½°. Calculate the tooth thickness and radius at a point on the involute which has a pressure angle of 25°.

Data :

$t_C = 7.98$ mm, $r_C = 88.9$ mm, $\phi_C = 14\frac{1}{2}°$, $\phi_D = 25°$

Solution :

We know that, $r_C \cos\phi_C = r_D \cos\phi_D$

$\therefore$ Radius of point D on the involute $r_D = \dfrac{r_C \cos\phi_C}{\cos\phi_D} = \dfrac{88.9 \times \cos 14.5}{\cos 25} = 94.966$ mm

Involute function at C is,

$$\text{inv}\,\phi_C = \tan\phi_C - \phi_C = \tan 14.5 - \frac{14.5 \times \pi}{180} = 5.545 \times 10^{-3}$$

Involute function at D is,

$$\text{inv}\,\phi_D = \tan\phi_D - \phi_D$$

$$= \tan 25 - \frac{25 \times \pi}{180} = 0.02998$$

Tooth thickness at point D, $t_D = 2\,r_D \left[\dfrac{t_C}{2r_C} + \text{inv}\,\phi_C - \text{inv}\,\phi_D\right]$

$$= 2 \times 94.966 \times \left[\frac{7.98}{2 \times 88.9} + 5.545 \times 10^{-3} - 0.02998\right] = 3.884 \text{ mm}$$

Example 6.31

The thickness of an involute gear tooth is 4.98 mm at a radius of 50.8 mm and a pressure angle of 20°. Calculate the tooth thickness on the base circle.

Data :

$t_C = 4.98$ mm, $r_C = 50.8$ mm, $\phi_C = 20°$, $r_D = r_B$

Solution :

Base circle radius $r_B = r_C \cos\phi_C$

$$= 50.8 \times \cos 20 = 47.736 \text{ mm}$$

Since the point B is on the base circle,

the pressure angle ϕ_B is zero.

$\therefore$ $\text{inv}\,\phi_B = \tan\phi_B - \phi_B = 0$

and $\text{inv}\,\phi_C = \tan\phi_C - \phi_C$

$$= \tan 20 - \frac{20 \times \pi}{180} = 0.0149$$

Tooth thickness of point B on the base circle is

$$t_B = 2\,r_B\left(\frac{t_C}{2\,r_C} + \text{inv}\,\phi_C - \text{inv}\,\phi_B\right)$$

$$= 2 \times 47.736 \times \left(\frac{4.98}{2 \times 50.8} + 0.0149 - 0\right) = 6.102 \text{ mm}$$

Example 6.32

The number of teeth on a 20° full depth involute gear is 22, and the module is 12 mm, calculate,

(i) Pitch circle radius

(ii) Thickness of tooth at the pitch circle

(iii) Base circle radius, and

(iv) Thickness of tooth at the base circle.

Data :

$\phi = 20°$, $z = 22$ teeth, $m = 12$ mm

Solution :

Pitch circle radius $r = \dfrac{mz}{2} = \dfrac{12 \times 22}{2} = 132$ mm

Circular pitch $\quad p = \dfrac{\pi d}{z} = \pi \text{m} = 12\,\pi$ mm

Tooth thickness at the pitch circle $t = \dfrac{p}{2} = \dfrac{12\pi}{2} = 6\,\pi$ mm

Involute function at the pitch circle inv $\phi = \tan\phi - \phi$

$$= \tan 20 - \frac{20 \times \pi}{180} = 0.0149$$

Base circle radius $r_B = r\cos\phi$

$$= 132 \cos 20 = 124.039 \text{ mm}$$

Involute function at the base circle inv $\phi_B = 0$ $\qquad (\because \phi_B = 0)$

Tooth thickness at base circle $t_B = 2r_B\left[\dfrac{t}{2r} + \text{inv}\,\phi - \text{inv}\,\phi_B\right]$

$$= 2 \times 124.039 \times \left[\frac{6\pi}{2 \times 132} + 0.0149 - 0\right] = 21.41 \text{ mm}$$

Example 6.33

A 10-mm module gear has 17 teeth, 20° pressure angle and addendum of 1 *m*. Find the thickness of the teeth at the base circle and at addendum circle. What is the pressure angle corresponding to the addendum circle?

Data :

$m = 10$ mm, $z = 17$, $\phi = 20°$, $a = 1m$

Solution :

Pitch circle radius $r = \dfrac{mz}{2} = \dfrac{10 \times 17}{2} = 85$ mm

Base circle radius $r_B = r \cos \phi$

$= 85 \cos 20 = 79.874$ mm

Addendum $a = 1\ m$

$= 1 \times 10 = 10$ mm

Addendum circle radius $r_a = r + a$

$= 85 + 10 = 95$ mm

We know that, $r \cos \phi = r_a \cos \phi_A$

i.e., $85 \cos 20 = 95 \cos \phi_A$

$\therefore$ Pressure angle at the addendum circle $\phi_a = 32.78°$

Involute function at pitch circle $\text{inv}\ \phi = \tan \phi - \phi$

$$= \tan 20 - \frac{20 \times \pi}{180} = 0.0149$$

Involute function at addendum circle $\text{inv}\ \phi_A = \tan \phi_A - \phi_A$

$$= \tan 32.78 - \frac{32.78 \times \pi}{180} = 0.07184$$

Circular pitch $p = \pi m = 10\ \pi$ mm

Tooth thickness at the pitch circle $t = \dfrac{p}{2} = \dfrac{10\pi}{2} = 5\ \pi$mm

Tooth thickness at addendum circle $t_A = 2\, r_A \left[\dfrac{t}{2r} + \text{inv}\,\phi - \text{inv}\,\phi_A\right]$

$$= 2 \times 95 \left[\frac{5\pi}{2 \times 85} + 0.0149 - 0.07184\right] = 6.737 \text{ mm}$$

Tooth thickness at the base circle $t_B = 2\, r_B \left[\dfrac{t}{2r} + \text{inv}\,\phi - \text{inv}\,\phi_B\right]$

The involute function at the base circle inv $\phi_B = 0$ $(\because \ \phi_B = 0)$

$$\therefore t_B = 2 \times 79.874 \left[\frac{5\pi}{2 \times 85} + 0.0149 - 0\right] = 17.14 \text{ mm}$$

Example 6.34

If the involute that is formed at the outline of the gear tooth are extended, they will intersect and the tooth becomes pointed. Determine the radius at which this occurs for a tooth which has a thickness of 6.65 mm at a radius of 100 mm and pressure angle 20°. **(VTU, Feb. 2006)**

Data:

$t_c = 6.65$ mm, $r_c = 100$ mm, $\phi_c = 20°$, $t_z = 0$ mm

Solution:

Involute function at C is, inv $\phi_c = \tan \phi_c - \phi_c = \tan 20 - 20 \times \dfrac{\pi}{180} = 0.0149$

Let r_z be the radius at which the tooth becomes pointed (tooth thickness is zero)

Tooth thickness at point Z, $t_z = 2\, r_z \left[\dfrac{t_c}{2 r_c} + \text{inv } \phi_c - \text{inv } \phi_z\right]$

i.e., $$0 = 2\, r_z \left[\frac{6.65}{2 \times 100} + 0.0149 - \text{inv } \phi_z\right]$$

or $$0 = \frac{6.65}{2 \times 100} + 0.0149 - \text{inv } \phi_z$$

$\therefore$ Involute function at Z, inv $\phi_z = 0.04815$

Also, $\text{inv } \phi_z = \tan \phi_z - \phi_z$

i.e., $$0.04815 = \tan \phi_z - \frac{\phi_D \times \pi}{180}$$

By trial and error method, pressure angle at Z, $\phi_z = 29°$

Also $r_c \cos \phi_c = r_z \cos \phi_z$

i.e., $100 \times \cos 20 = r_z \times \cos 29$

$\therefore$ Radius at which the tooth become pointed, $r_z = 107.44$ mm

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Example 6.35

Find the thickness at the addendun and base circles of a tooth of 3 mm module. The number of teeth on gear is 30 and the pressure angle is 20°. The tooth thickness at the pitch circle equal to half the circular pitch and the addendum is one module. The gear tooth thickness becomes smaller as the radius increases from the base circle radius. Determine the addendum circle radius if the tooth thickness is zero.

Data:

ϕ=20°, $z = 30$ teeth, $m = 3$ mm, $a = 1\, m = 3$ mm, $t = p/2$

Solution:

Pitch circle radius $r = \dfrac{mz}{2} = \dfrac{3 \times 30}{2} = 45$ mm

Circular pitch $p = \dfrac{\pi d}{z} = \pi m = \pi \times 3 = 3\pi$ mm

Tooth thickness at the pitch circle $t = \dfrac{p}{2} = \dfrac{3\pi}{2} = 1.5\,\pi \text{mm} = 4.7124$ mm

Involute function at the pitch circle inv $\phi = \tan\phi - \phi$

$$= \tan 20 - \frac{20 \times \pi}{180} = 0.0149$$

Base circle radius $r_B = r \cos\phi = 45 \cos 20 = 42.286$ mm

Involute function at the base circle inv $\phi_B = 0$

$\therefore$ Tooth thickness at base circle $t_B = 2\, r_B \left[\dfrac{t}{2r} + \text{inv}\ \phi - \text{inv}\ \phi_B\right]$

$$= 2 \times 42.286 \left[\frac{4.7124}{2 \times 45} + 0.0149 - 0\right] = 5.688 \text{ mm}$$

Addendum circle radius $r_a = r + a = 45 + 3 = 48$ mm

Also $r \cos\phi = r_a \cos\phi_a$

i.e., $45 \cos 20 = 48 \cos\phi_a$

$\therefore$ Pressure angle at the addendum circle $\phi_a = 28.2414°$

Involute function at addendum circle inv $\phi_a = \tan\phi_a - \phi_a$

$$= \tan 28.2414 - \frac{28.2414 \times \pi}{180} = 0.04422$$

Tooth thickness at addendum circle $t_a = 2\, r_a \left[\dfrac{t}{2r} + \text{inv}\ \phi - \text{inv}\ \phi_a\right]$

$$= 2 \times 48 \left[\frac{4.7124}{2 \times 45} + 0.0149 - 0.04422\right] = 2.2118 \text{ mm}$$

Let r_z be the radius at which the tooth becomes pointed (tooth thickness is zero)

$$\therefore \quad 0 = 2\, r_z \left[\frac{t}{2r} + \text{inv}\ \phi - \text{inv}\ \phi_z\right]$$

or $\dfrac{t}{2r} + \text{inv}\ \phi - \text{inv}\ \phi_z = 0$

i.e., $\dfrac{4.7124}{2 \times 45} + 0.0149 - \text{inv}\,\phi_z = 0$

$$\therefore \qquad \text{inv}\,\phi_z = 0.06726 = \tan\phi_z - \frac{\phi_z \times \pi}{180}$$

By trial and error method,

Pressure angle when the tooth thickness is zero, $\phi_z = 32.13°$

Also, $\quad r\cos\phi = r_z\cos\phi_z$

$$45\cos 20 = r_z\cos 32.13$$

Radius at which the tooth thickness is zero $r_z = 49.934$ mm

Determination of backlash

In practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors or thermal expansion. For a set of mating gears, backlash can be provided either mounting the gears at a center distance greater than the theoretical or by feeding the cutter deeper than standard. Fig. 6.15*a* shows two standard gears meshing at standard center distance *c*.

Let $\quad r_1$ = Standard pitch circle radius of pinion

r_2 = Standard pitch circle radius of gear

$\therefore$ Standard center distance $c = r_1 + r_2$ ...... (1)

Fig. 6.15*b* shows the condition where the two gears have been pulled apart at a distance Δc to give a new center distance c'. The line of action now crosses the line of centers at a new pitch point p'.

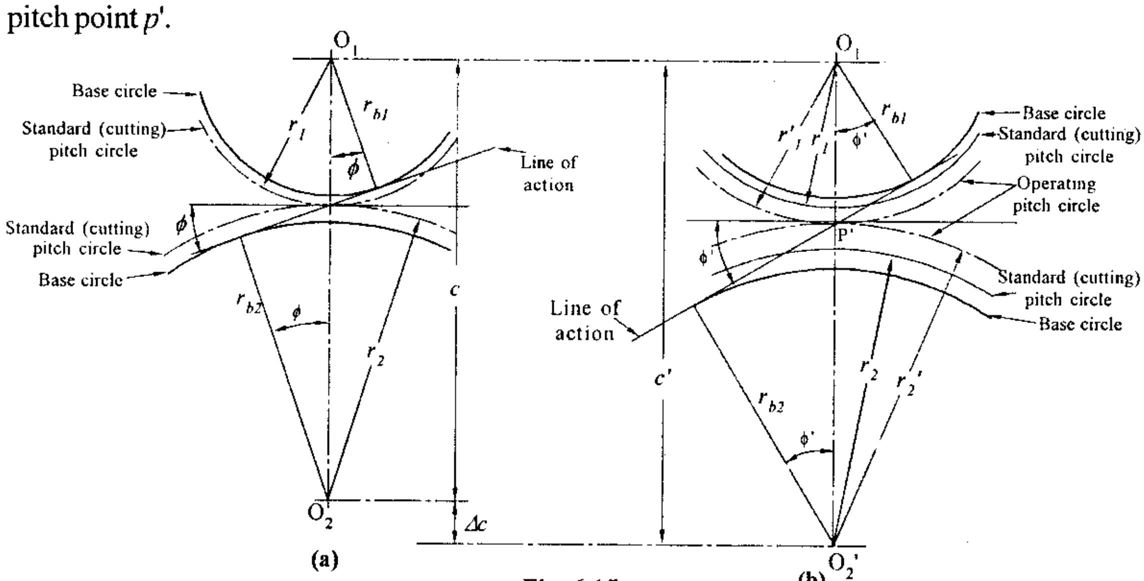

Fig. 6.15

Let r_1' = Operating pitch circle radius of pinion

r_2' = Operating pitch circle radius of gear

$\therefore$ Operating center distance $c' = r_1' + r_2'$ ...... (2)

Also $$\frac{r_1}{r_1'} = \frac{r_2}{r_2'} = \frac{c}{c'}$$ ...... (3)

We know that,

$$c' \cos \phi' = c \cos \phi$$

Where ϕ = Cutting pressure angle

ϕ' = Operating pressure angle

$$\therefore c' = \frac{c \cos\phi}{\cos\phi'}$$

charge in center distance $\Delta c = c' - c$

$$= \frac{c\cos\phi}{\cos\phi'} - c = c\left(\frac{\cos\phi}{\cos\phi'} - 1\right)$$ ...... (4)

On the operating pitch circle, the sum of the tooth thickness plus the backlash must be equal to the circular pitch.

i.e., $t_1' + t_2' + B = p'$ ...... (5)

Where t' = Tooth thickness on operating pitch circle

B = Backlash

p' = Operating circular pitch.

Let t = Tooth thickness on standard pitch circle $= \frac{p}{2}$

p = standard circular pitch

By involutometry, the thickness of the tooth on the operating pitch circle is,

$$t_1' = 2\, r_1' \left[\frac{t}{2r_1} + \text{inv}\,\phi - \text{inv}\,\phi'\right]$$ ...... (6)

and $$t_2' = 2\, r_2' \left[\frac{t}{2r_2} + \text{inv}\,\phi - \text{inv}\,\phi'\right]$$ ...... (7)

Substitute the values of t_1' and t_2' in equation (5), we get

$$2\, r_1' \left[\frac{t}{2r_1} + \text{inv}\,\phi - \text{inv}\,\phi'\right] + 2\, r_2' \left[\frac{t}{2r_2} + \text{inv}\,\phi - \text{inv}\,\phi'\right] + B = p'$$

$$t\left(\frac{r_1'}{r_1}+\frac{r_2'}{r_2}\right)+2\,\text{inv}\,\phi(r_1'+r_2')-2\,\text{inv}\,\phi'\,(r_1'+r_2')+B=p'$$

$$t\left(\frac{c'}{c}+\frac{c'}{c}\right)+2\,c'\,\text{inv}\,\phi-2\,c'\,\text{inv}\,\phi'+B=p' \qquad \left\{\because \frac{r_1'}{r_1}=\frac{r_2'}{r_2}=\frac{c'}{c},\text{ and } r_1'+r_2'=c'\right\}$$

$$B = p'-2\,t\times\frac{c'}{c}+2\,c'\,(\text{inv}\,\phi'-\text{inv}\,\phi)$$

$$= \frac{2\pi r_1'}{z_1}-2\times\frac{2\pi r_1}{2z_1}\times\frac{c'}{c}+2\,c'\,(\text{inv}\,\phi'-\text{inv}\,\phi)$$

$$\left(\because p'=\frac{2\pi r_1'}{z_1}\text{ and } t=\frac{p}{2}=\frac{2\pi r_1}{2z_1}\right)$$

$$= \frac{2\pi}{z_1}\left(r_1'-r_1\times\frac{c'}{c}\right)+2\,c'\,(\text{inv}\,\phi'-\text{inv}\,\phi)$$

$$= \frac{2\pi}{z_1}\left(r_1'-r_1\times\frac{r_1'}{r_1}\right)+2\,c'\,(\text{inv}\,\phi'-\text{inv}\,\phi) \qquad \left(\because \frac{c'}{c}=\frac{r_1'}{r_1}\right)$$

$\therefore$ Backlash $\quad B = 2\,c'\,(\text{inv}\,\phi'-\text{inv}\,\phi)$ ......(8)

Example 6.36

A 2.5 mm module, 20° pinion with 36 teeth drives a gear with 60 teeth. If the center distance is increased by 0.65 mm, calculate,

i) The radii of the operating pitch circles,

ii) The operating pressure angle, and

iii) Backlash produced.

Data :

$m = 2.5$ mm, $\phi = 20°$, $z_1 = 36$ teeth, $z_2 = 60$ teeth, $\Delta c = 0.65$ mm

Solution :

Standard pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{2.5\times 36}{2} = 45$ mm

Standard pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{2.5\times 60}{2} = 75$ mm

Standard center distance $c = r_1 + r_2 = 45 + 75 = 120$ mm

Operating center distance $c' = c + \Delta c = 120 + 0.65 = 120.65$ mm

We know that, $\frac{r_1}{r_1'} = \frac{c}{c'}$

i.e., $\frac{45}{r_1'} = \frac{120}{120.65}$

∴ Operating pitch circle radius of pinion $r_1' = 45.244$ mm

Operating pitch circle radius of gear $r_2' = c' - r_1'$

$= 120.65 - 45.244 = 75.406$ mm

Also $c \cos\phi = c' \cos\phi'$

i.e., $120 \cos 20 = 120.65 \cos\phi'$

∴ Operating pressure angle $\phi' = 20.83°$

Backlash $B = 2c' (\text{inv } \phi' - \text{inv } \phi)$

where $\text{inv } \phi' = \tan\phi' - \phi'$

$= \tan 20.83 - \frac{20.83 \times \pi}{180} = 0.0169$

and $\text{inv } \phi = \tan\phi - \phi$

$= \tan 20 - 20 \times \frac{\pi}{180} = 0.0149$

∴ Backlash $B = 2 \times 120.65\,(0.0169 - 0.0149) = 0.4826$ mm

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Example 6.37

The following data refer to two mating involute geares of 20° pressure angle :

Number of teeth on pinion = 20, Gear ratio = 2, Module = 12 mm

If the center distance between the gears is increased by 2 mm., find the backlash between the gears. **(VTU, Aug, 2000)**

Data :

$\phi = 20°$, $z_1 = 20$ teeth, $i = 2$, $m = 12$ mm, $\Delta c = 2$ mm

Solution :

Number of teeth on gear $z_2 = iz_1 = 2 \times 20 = 40$

Pitch circle radius of pinion $r_1 = \frac{mz_1}{2} = \frac{12 \times 20}{2} = 120$ mm

Pitch circle radius of gear $r_2 = \frac{mz_2}{2} = \frac{12 \times 40}{2} = 240$ mm

Center distance $c = r_1 + r_2 = 120 + 240 = 360$ mm

Operating center distance $c' = c + \Delta c = 360 + 2 = 362$ mm

We know that $c \cos \phi = c' \cos \phi'$

i.e., $360 \times \cos 20 = 362 \times \cos \phi'$

$\therefore$ Operating pressure angle $\phi' = 20.852°$

Backlash $B = 2c'(\text{inv } \phi' - \text{inv } \phi)$

where $\text{inv } \phi' = \tan \phi' - \phi' = \tan 20.825 - \frac{20.852 \times \pi}{180} = 0.016968$

$\text{inv } \phi = \tan \phi - \phi = \tan 20 - \frac{20 \times \pi}{180} = 0.0146$

$\therefore$ Backlash $B = 2 \times 362\,(0.016968 - 0.0149) = 1.494$ mm

Example 6.38

A 20° involute pinion having 24 teeth drives a gear of 60 teeth. The module is 3 mm and the addendum is one module. (a) Calculate the length of action and contact ratio if the gears mesh with zero backlash. (b) If the center distance is increased by 0.5 mm, calculate the radii of the operating pitch circles, the operating pressure angle and the backlash produced.

Data:

$\phi = 20°$, $z_1 = 24$ teeth, $z_2 = 60$ teeth, $m = 3$ mm, $a_1 = a_2 = 1\,m = 1 \times 3 = 3$ mm, $\Delta c = 0.5$ mm

Solution:

Pitch circle radius of pinion $r_1 = \frac{m z_1}{2} = \frac{3 \times 24}{2} = 36$ mm

Pitch circle radius of gear $r_2 = \frac{m z_2}{2} = \frac{3 \times 60}{2} = 90$ mm

Addendum circle radius of pinion $r_{a1} = r_1 + a_1 = 36 + 3 = 39$ mm

Addendum circle radius of gear $r_{a2} = r_2 + a_2 = 90 + 3 = 93$ mm

$$\text{Path of contact} = \sqrt{r_{a2}^2 - r_2^2 \cos^2 \phi} + \sqrt{r_{a1}^2 - r_1^2 \cos^2 \phi} - (r_1 + r_2) \sin \phi$$

$$= \sqrt{93^2 - 90^2 \cos^2 20} + \sqrt{39^2 - 36^2 \cos^2 20} - (36 + 90) \sin 20$$

$$= 14.9966 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Path of contact}}{p \cos \phi} = \frac{\text{Path of contact}}{\pi m \cos \phi} \qquad (\because p = \pi m)$$

$$= \frac{14.9966}{\pi \times 3 \times \cos 20} = 1.6933$$

b) Correct center distance $c = r_1 + r_2 = 36 + 90 = 126$ mm

Operating center distance $c' = c + \Delta c = 126 + 0.5 = 126.5$ mm

We know that, $\dfrac{r}{r'_1} = \dfrac{c}{c'}$

i.e., $\dfrac{36}{r'_1} = \dfrac{126}{126.5}$

$\therefore$ Operating pitch circle radius of pinion $r'_1 = 36.1429$ mm

Operating pitch circle radius of gear $r'_2 = c' - r'_1 = 126.5 - 36.1429 = 90.3571$ mm

Also $c \cos \phi = c' \cos \phi'$

$126 \cos 20 = 126.5 \cos \phi'$

$\therefore$ Operating pressure angle $\phi' = 20.6132°$

Backlash $B = 2c'(\text{inv } \phi' - \text{inv } \phi)$

where $\text{inv } \phi' = \tan \phi' - \phi' = \tan 20.6132 - 20.6132 \times \dfrac{\pi}{180} = 0.01637$

$\text{inv } \phi = \tan \phi - \phi = \tan 20 - 20 \times \dfrac{\pi}{180} = 0.0149$

$\therefore$ Backlash $B = 2 \times 126.5\,(0.01637 - 0.0149)$

$= 0.3719$ mm

REVIEW QUESTIONS

1. What are toothed gears? State their uses.
2. What are the advantages and disadvantages of gear drives?
3. How are gears classified?
4. State the law of gearing.
5. What are the common curves used for tooth profile?
6. Discuss the relative merits and demerits of involute curve and cycloidal curve for the profiles of gear tooth.
7. What are the general characteristics of spur gearing?
8. Draw a neat sketch of the tooth profile for a spur gear and explain the various terms connected with it.
9. Define the following:
   i) Pitch circle diameter, (ii) Circular pitch, (iii) Module, (iv) Addendum, (v) Dedendum, (vi) Pressure angle.
10. Define the following :
    i) Path of contact, (ii) Arc of contact, (iii) Contact ratio.

11. Derive the formula for length of arc of contact for two meshing spur gears of involute profile.
12. Explain the phenomenon of interference. State the necessary condition for no interference.
13. Describe the various methods of avoiding interference.
14. Derive an expression for minimum number of teeth necessary for a pinion to avoid interference.
15. Derive an expression for the length of path of contact for a pinion on rack.
16. What is involutometry? Derive an expression for the tooth thickness at any point on the involute, if the tooth thickness at some other point is known.
17. What is backlash? Derive an expression for backlash if the center distance is pulled apart a distance Δc.

EXERCISE – 6

1. A pinion of module pitch 4 mm have 24 teeth and the gear ratio is 1.25. The pressure angle is 20° and each wheel has a standard addendum of 1 module. Find the length of arc of contact and the maximum velocity of sliding if the speed of the pinion is 400 rpm.

   [**Ans.** 19 mm, 736 mm/sec)

2. Two gear wheels mesh externally and are to give a velocity ratio of 1.5. The pinion has 14 teeth and the pressure angle is 14.5°. Find the maximum addenda on the pinion and gear wheel to avoid interference if pitch in module is 6 mm. Also find the velocity of sliding of teeth on either side of pitch point.

   [**Ans.** 6.42 mm, 3.42 mm, 550.6 mm/sec, 825.9 mm/sec]

3. The following data relate to a pair of involute spur gears in mesh. Number of teeth on pinion = 22, Gear ratio = 2, Pressure angle = 20°, Module = 4 mm, Pitch line velocity of pinion = 1 m/sec. Determine the velocity of sliding at the approach, recess and at the pitch point.

   [**Ans.** 348.5 mm/sec, 318.2 mm/sec, 0]

4. The pitch circle diameter of pinion is 100 mm. The number of teeth on pinion and gear are 25 and 50 respectively. The pressure angle is 14.5° and the addendum is 0.32 of the circular pitch. Find the length of path of contact. [**Ans.** 24.36 mm]

5. A pinion of module pitch 6 mm have 20 teeth and the gear ratio is 1.5. The addendum on both wheels is 1/4 of the circular pitch. The angle of obliquity is 20°. Find,
   a) path of approach,
   b) arc of approach,
   c) path of recess and,
   d) arc of recess. [**Ans.** 11.83 mm, 12.6 mm, 11.2 mm, 11.9 mm]

6. Two mating gears have 30 and 40 involute teeth of module 12 mm and 20° obliquity. The addendum on each wheel is to be made such that a length of path of contact on each side of the pitch point has half the maximum possible length. Determine the addendum for each gear wheel and the length of path of contact. [**Ans.** 17.8 mm, 12.2 mm, 71.8 mm]

7. Determine the number of pairs of teeth in contact at a given instant if two equal involute gears of 20 teeth and pressure angle 20° have addendum of 0.8 module. [**Ans.** 1.289]

8. A pinion with 25 involute teeth of 150 mm of pitch circle diameter drives a rack. The addendum of the pinion and rack is 6 mm. Find the least pressure angle which can be used if undercutting of the teeth is to be avoided. [**Ans.** 16.43°]

9. Two equal gear wheels of 200 mm pitch circle diameter are in mesh. The module pitch is 4 mm and the pressure angle is 20°. The teeth are of involute form. Calculate the least addendum necessary if it is to be ensured that two pairs of teeth are always in contact. [**Ans.** 4.6 mm]

10. Two wheels with standard involute teeth of 3 mm module are to gear together with a velocity ratio of 3, the pressure angle being 14.5°. Find the least number of teeth on gear and pinion, if the interference is to be avoided. The standard addendum is 0.82 of the module.

    [**Ans.** 69 teeth, 23 teeth]

11. A pinion having 17 teeth drives a gear having 49 teeth. The pressure angle is 20°, module is 6 mm and the addendum on pinion and gear wheel = 1 module. Calculate,

    (i) Length of path of contact (ii) Arc of contact, and (iii) Contact ratio (July 2006)
    [**Ans:** 28.9245 mm, 30.78 mm, 31.6329]

12. A 3 mm module, 20° pinion with 24 teeth drives a 56 tooth gear. Determine the addendum circle radii so that the addendum circle of each gear passes through the interference point of the other. [**Ans:** 41.84 mm, 97.63 mm]

13. The thickness of an involute gear tooth is 10 mm at a radius of 100 mm and a pressure angle of $14\frac{1}{2}$°. Calculate the pressure angle and tooth thickness at a point on the involute that has a radius of 110 mm. [**Ans.** 28.34°, 2.38 mm]

14. A 4 mm module, $14\frac{1}{2}$° pinion of 20 teeth drives a gear of 60 teeth. If the center distance is increased by 0.6 mm, calculate the radii of the operating pitch circles, operating pressure angle, and the backlash. [**Ans.** 40 mm, 80 mm, 15.31°, 0.32 mm]

15. Two mating involute gears of 20° pressure angle have a gear ratio of 2. The number of teeth on pinion is 20. The speed of pinion is 250 rpm. Take module as 12 mm. If the addendum on each wheel is such that the path of approach and path of recess on each side are of half the maximum possible length each, find: i) The addendum of pinion and gear ii) The length of arc of contact. **(VTU, Jan 2008)**